Review Sheet

Operations with Rational Numbers

Adding Rational Numbers

Rules for Adding Fractions

1. Find a common denominator. The least common denominator (LCD) is best, but not necessary.
2. Add the numerators. Keep the denominator the same.
3. Simplify, if necessary. Rename, if necessary.

Rules for Adding Decimals

1. Line up the decimals.
2. Add like we do with whole numbers. The decimal drops straight down.

Rules for Adding Integers

- positive + positive = positive
- negative + negative = negative

Add the absolute values of the two numbers.

- positive + negative = ?
- negative + positive = ?

Subtract the absolute values of the two numbers. Take the sign of the larger number.

Subtracting Rational Numbers

Rules for Subtracting Fractions

1. Find a common denominator. The least common denominator (LCD) is best, but not necessary.
2. Subtract the numerators. If we have mixed numbers, we may have to borrow. Keep the denominator the same.
3. Simplify, if necessary. Rename, if necessary.

Rules for Subtracting Decimals

1. Line up the decimals.
2. Subtract like we do with whole numbers. The decimal drops straight down.

Rules for Subtracting Integers

1. Change minus sign to add the opposite (– to +).
2. Follow the rules for adding integers.

Examples:

1. $(-11.4) + (7.95) = \boxed{-3.45}$

$$\begin{array}{r} {}^{0}\cancel{1}{}^{10}\cancel{3}{}^{1} \\ 11.40 \\ -\ 7.95 \\ \hline 3.45 \end{array}$$

2. $\frac{8}{9} + \left(-1\frac{2}{3}\right) = \boxed{-\frac{7}{9}}$

$$1\frac{2}{3} \overset{\times 3}{=} 1\frac{6}{9} = \frac{15}{9}$$

$$-\frac{8}{9} = -\frac{8}{9} = -\frac{8}{9}$$

$$\frac{7}{9}$$

3. $-17.2 \ominus 3.87 =$

$-17.2 \oplus 3.87 = \boxed{-21.07}$

$$\begin{array}{r} {}^{1}{}^{1} \\ 17.20 \\ +\ 3.87 \\ \hline 21.07 \end{array}$$

4. $-2\frac{3}{5} \ominus \frac{1}{3} =$

$-2\frac{3}{5} \oplus -\frac{1}{3} = \boxed{-2\frac{14}{15}}$

$$2\frac{3}{5} \overset{\times 3}{=} 2\frac{9}{15}$$

$$+\ \frac{1}{3} \overset{\times 5}{=} +\frac{5}{15}$$

$$2\frac{14}{15}$$

Multiplying Rational Numbers

Rules for Multiplying Fractions

1. Change mixed numbers and whole numbers to improper fractions.
2. Multiply the numerators. Multiply the denominators. Cross cancel, if possible.
3. Simplify, if necessary. Rename, if necessary.

Rules for Multiplying Decimals

1. Multiply as if there are no decimals (like we are multiplying two whole numbers).
2. Count the number of total decimal places in the factors. This sum is the number of decimal places in our product (the answer to a multiplication problem).

Rules for Multiplying Integers

- positive × positive = positive
- negative × negative = positive
- positive × negative = negative
- negative × positive = negative

Dividing Rational Numbers

Rules for Dividing Fractions

1. Change mixed numbers and whole numbers to improper fractions.
2. Keep the first fraction the same. Change the division sign (÷) to a multiplication sign (×). Find the reciprocal of the second fraction (flip the fraction).
3. Multiply the numerators. Multiply the denominators. Cross cancel, if possible.
4. Simplify, if necessary. Rename, if necessary.

Rules for Dividing Decimals

1. Move the decimal in the divisor (the one doing the dividing; the one outside the "house") as many places to the right as necessary to make a whole number. Move the decimal in the dividend (the one being divided; the one inside the "house") the same number of places to the right.
2. Divide like we do with whole numbers. The decimal goes straight up.

Rules for Dividing Integers

- positive ÷ positive = positive
- negative ÷ negative = positive
- positive ÷ negative = negative
- negative ÷ positive = negative

Examples:

1. $15.2 \times (-0.18) = \boxed{-2.736}$

$$\begin{array}{r} {}^{4}{}^{1}15.2 \leftarrow 1 \\ \times\ 0.18 \leftarrow +2 \\ \hline 1216 \\ +1520 \\ \hline 2736 \end{array}$$

3 decimal places

2. $-\frac{3}{7} \times \left(-2\frac{4}{5}\right) = \boxed{1\frac{1}{5}}$

$$-\frac{3}{\cancel{7}_{1}} \times \left(-\frac{\cancel{14}^{2}}{5}\right) = \frac{6}{5} = 1\frac{1}{5}$$

3. $-5 \div \left(-1\frac{2}{3}\right) = \boxed{3}$

$$-\frac{5}{1} \div \left(-\frac{5}{3}\right) =$$

$$-\frac{\cancel{5}^{1}}{1} \times \left(-\frac{3}{\cancel{5}_{1}}\right) = \frac{3}{1} = 3$$

4. $-101.088 \div 7.02 = \boxed{-14.4}$

$$\begin{array}{r} 14.4 \\ 7.02\overline{)101.088} \\ 702\downarrow \\ \hline 3088 \\ 2808\downarrow \\ \hline 2808 \\ -2808 \\ \hline \end{array}$$

Writing and Solving Proportions

A proportion is an equation showing that two ratios are equal because there are relationships between the two ratios. There are several different ways to write proportions and two different ways to solve proportions.

Example:

In an official United States flag, the ratio of the length of the flag to the width must be 19 to 10. A company makes a souvenir U.S. flag that is 30 cm wide. In order for it to be an official U.S. flag, how long must the souvenir flag be?

(Source: *The Flag Book of the United States)*

We can use proportions to solve problems like the one in the example above. The first step is to write a proportion that shows the relationships between the two ratios. Some common element must tie the numerators together. Another common element must tie the denominators together. Also, each ratio must have a relationship.

- Both numerators could have the relationship, "length," while both denominators could have the relationship, "width." One ratio could have the relationship, "souvenir flag," while the other ratio could have the relationship, "official flag."

$$\frac{\text{length} \rightarrow}{\text{width} \rightarrow} : \frac{\underline{\text{souvenir flag}} \downarrow \text{length of souvenir}}{\text{width of souvenir}} = \frac{\underline{\text{official flag}} \downarrow \text{official length}}{\text{official width}}$$

OR

- Both numerators could have the relationship, "souvenir flag," while both denominators could have the relationship, "official flag." One ratio could have the relationship, "length," while the other ratio could have the relationship, "width."

$$\frac{\text{souvenir flag} \rightarrow}{\text{official flag} \rightarrow} : \frac{\underline{\text{length}} \downarrow \text{length of souvenir}}{\text{official length}} = \frac{\underline{\text{width}} \downarrow \text{width of souvenir}}{\text{official width}}$$

Using the templates above, we can write these two proportions, $\frac{n}{30\text{ cm}} = \frac{19}{10}$ or $\frac{n}{19} = \frac{30\text{ cm}}{10}$. In both proportions the n represents the length of the souvenir flag.

The second step to solve a problem using a proportion is to solve the proportion. For the first proportion, we can use equivalent fractions to solve the proportion because 10 is a factor of 30. Below, we see that, since 10 x 3 = 30 cm, we can multiply 19 x 3 to get n = 57 cm.

$$\frac{n = 57 \text{ cm}}{30 \text{ cm}} = \frac{= 3 \times 19}{= 3 \times 10}$$

For the second proportion, we can solve the proportion in two different ways. As in the first proportion, the first way we can solve the proportion is to use equivalent fractions. However, because 10 is not a factor of 19, first we need to simplify the ratio on the right by dividing the numerator and denominator by 10.

$$\frac{30 \text{ cm} \div 10 =}{10 \div 10 =} = \frac{3 \text{ cm}}{1}$$

Below, we now see that 1 is a factor of 19. Since 1 x 19 = 19, we can multiply 3 cm x 19 to get n = 57 cm.

$$\frac{n = 57 \text{ cm}}{19} = \frac{= 19 \times 3 \text{ cm}}{= 19 \times 1}$$

For the second proportion, the second way that we can solve the proportion is to use cross products.

$$\frac{n}{30} = \frac{19}{10}$$

$$10n = 19(30)$$

$$\frac{10n}{10} = \frac{570}{10}$$

$$n = 57 \text{ cm}$$