Eighth Grade Summer Review Sheet

Adding Integers

Same Sign: The sum of two positive numbers is positive. The sum of two negative numbers is negative.

Examples:

3 + 5 = 8	-3 + (-5) = -8
Both integers are positive.	Both integers are negative.
The sum is positive.	The sum is negative.

Different Signs:

Step 1: Find the absolute value of each number (The absolute value is the distance from the integer to 0 on the number line. For example, the absolute value of -23, labeled $|-23|$, is 23, since -23 is 23 units away from 0. The absolute value of 8, labeled $|8|$, is 8, since 8 is 8 units away from 0.).

Step 2: Subtract the lesser absolute value from the greater.

Step 3: The sum has the sign of the integer with the greater absolute value.

Examples:

1. Find -3 + 5.

Step 1: $|-3| = 3$ and $|5| = 5$ ← **Find the absolute value of each integer.**

Step 2: 5 – 3 = 2 ← **Subtract 3 from 5 because $|-3| < |5|$.**

Step 3: -3 + 5 = 2 ← **The sum has the same sign as 5 because 5 has the greater absolute value.**

2. Find 5 + (-9).

Step 1: $|5| = 5$ and $|-9| = 9$ ← **Find the absolute value of each integer.**

Step 2: 9 – 5 = 4 ← **Subtract 9 from 5 because $|-9| < |5|$.**

Step 3: 5 + -9 = -4 ← **The sum has the same sign as -9 because 9 has the greater absolute value.**

Subtracting Integers

Step 1: Change the minus sign (–) to add the opposite (+⁻).

Example: 4 – 6 becomes 4 + (-6) because the opposite of 6 is -6.

-9 – (-7) becomes -9 + 7 because the opposite of -7 is 7.

Step 2: Add the integers following the rules for adding integers (see page 1).

Examples:

1. Find -5 – 10.

Step 1: -5 – 10 = -5 + (-10)

Step 2: -5 + (-10) = -15

2. Find -6 – (-11)

Step 1: -6 – (-11) = -6 + 11

Step 2: -6 + 11 = 5

Multiplying and Dividing Integers

Same Sign: The product or quotient of two integers with the same sign is always positive.

Different Signs: The product or quotient of two integers with different signs is always negative.

Note: The signs for multiplication include $\times$, $\bullet$, and parentheses.

The signs for division include $\div$, $\overline{)}$, and the fraction bar.

Examples:

1. Find -5(3).

-5(3) = -15 because the two integers have different signs, so the product is negative.

2. Find $\frac{-25}{-5}$.

$\frac{-25}{-5} = 5$ because the two integers have the same sign, so the quotient is positive.

Writing and Solving Proportions

A proportion is an equation showing that two ratios are equal because there are relationships between the two ratios. There are several different ways to write proportions and two different ways to solve proportions.

Example:

In an official United States flag, the ratio of the length of the flag to the width must be 19 to 10. A company makes a souvenir U.S. flag that is 30 cm wide. In order for it to be an official U.S. flag, how long must the souvenir flag be?

(Source: *The Flag Book of the United States)*

We can use proportions to solve problems like the one in the example above. The first step is to write a proportion that shows the relationships between the two ratios. Some common element must tie the numerators together. Another common element must tie the denominators together. Also, each ratio must have a relationship.

- Both numerators could have the relationship, "length," while both denominators could have the relationship, "width." One ratio could have the relationship, "souvenir flag," while the other ratio could have the relationship, "official flag."

$$\frac{\text{length} \rightarrow}{\text{width} \rightarrow} : \frac{\underset{\downarrow}{\text{length of souvenir}}}{\text{width of souvenir}} = \frac{\underset{\downarrow}{\text{official length}}}{\text{official width}}$$

OR

- Both numerators could have the relationship, "souvenir flag," while both denominators could have the relationship, "official flag." One ratio could have the relationship, "length," while the other ratio could have the relationship, "width."

$$\frac{\text{souvenir flag} \rightarrow}{\text{official flag} \rightarrow} : \frac{\underset{\downarrow}{\text{length of souvenir}}}{\text{official length}} = \frac{\underset{\downarrow}{\text{width of souvenir}}}{\text{official width}}$$

Using the templates above, we can write these two proportions, $\frac{n}{30\text{ cm}} = \frac{19}{10}$ or $\frac{n}{19} = \frac{30\text{ cm}}{10}$. In both proportions the n represents the length of the souvenir flag.

The second step to solve a problem using a proportion is to solve the proportion. For the first proportion, we can use equivalent fractions to solve the proportion because 10 is a factor of 30. Below, we see that, since 10 x 3 = 30 cm, we can multiply 19 x 3 to get n = 57 cm.

$$\frac{n = 57 \text{ cm}}{30 \text{ cm}} = \frac{= 3 \times 19}{= 3 \times 10}$$

For the second proportion, we can solve the proportion in two different ways. As in the first proportion, the first way we can solve the proportion is to use equivalent fractions. However, because 10 is not a factor of 19, first we need to simplify the ratio on the right by dividing the numerator and denominator by 10.

$$\frac{30 \text{ cm} \div 10 =}{10 \div 10 =} = \frac{3 \text{ cm}}{1}$$

Below, we now see that 1 is a factor of 19. Since 1 x 19 = 19, we can multiply 3 cm x 19 to get n = 57 cm.

$$\frac{n = 57 \text{ cm}}{19} = \frac{= 19 \times 3 \text{ cm}}{= 19 \times 1}$$

For the second proportion, the second way that we can solve the proportion is to use cross products.

$$\frac{n}{30} = \frac{19}{10}$$

$$10n = 19(30)$$

$$\frac{10n}{10} = \frac{570}{10}$$

$$n = 57 \text{ cm}$$

QAR: Literal

1. Define proportion.

an equation showing that two ratios are equal because there are relationships between the two ratios.

QAR: Literal

2. What are the two ways that we can solve a proportion?

1) Use equivalent fractions

2) Use cross products

QAR: Inference

3. Kiana wants to make an official souvenir United States flag. She wants the width of her souvenir flag to be 40 cm. How long should her souvenir flag be?

Write proportion

official → 19 / 10 = n / 40 ← souvenir

length : width

Solve proportion

$\frac{19}{10} = \frac{n}{40}$

$10n = 19(40)$

$\frac{10n}{10} = \frac{760}{10}$

$n = 76$ cm

The length of her souvenir flag would be 76 cm.

QAR: Evaluation

4. Kumu Kimo calculates that he needs 2 yards of material for every 3 girls in order to make the pāʻū skirts. If there are 53 girls in Hawaiian Ensemble, about how many yards of material would Kumu Kimo need?

Write proportion

ratio → 2 / 3 = n / 53 ← actual

yards : girls

Solve proportion

$\frac{2}{3} = \frac{n}{53}$

$3n = 2(53)$

$\frac{3n}{3} = \frac{106}{3}$

$n = 35.\overline{3}$ yards.

Kumu Kimo should buy 36 yards of material

QAR: Evaluation

5. Mrs. Ah Chong wants to buy a dress that costs $55 from Macy's. She has a coupon for 20% off. How much discount would Mrs. Ah Chong get if she uses the coupon?

Write proportion

percent → 20 / 100 = n / 55 ← actual

part : whole

Solve proportion

$\frac{20}{100} = \frac{n}{55}$

$100n = 20(55)$

$\frac{100n}{100} = \frac{1100}{100}$

$n = \$11$

Mrs. Ah Chong's discount would be $11.