



Polynomial Functions

Who uses this?

Doctors can use polynomials to model blood flow.

A **monomial** is a number or a product of numbers and variables with whole number exponents. A **polynomial** is a monomial or a sum or difference of monomials. Each monomial in a polynomial is a term. Because a monomial has only one term, it is the simplest type of polynomial.

Polynomials have no variables in denominators or exponents, no roots or absolute values of variables have whole number exponents.

Polynomials:	$3x^4$	$2z^{12} + 9z^3$	$\frac{1}{2}a^7$	$0.15x^{101}$	$3t^2 - t^3$
Not Polynomials:	$3x$	$ 2b^3 - 6b $	$\frac{8}{5y^2}$	$\frac{1}{2}\sqrt{x}$	$m^{0.75} - m$

The **degree of a monomial** is the sum of the exponents of the variables.

Example 1 – Identifying the Degree of a Monomial

Identify the degree of each monomial.

A. x^4

B. 12

C. $4a^2b$

D. x^3y^4z

The **degree of a polynomial** is given by the term with the greatest degree. A polynomial with one variable is in standard form when its terms are written in descending order by degree. So, in standard form, the degree of the first term indicates the degree of the polynomial, and the **leading coefficient** is the coefficient of the first term.

Standard Form

Leading coefficient → $5x^3 + 8x^2 + 3x - 17$ ← **Degree of polynomial**

Degree of term: 3 2 1 0

A polynomial can be classified by its number of terms. A polynomial with two terms is called a **binomial**, and a polynomial with three terms is called a **trinomial**. A polynomial can also be classified by its degree.

Classifying Polynomials by Degree		
Name	Degree	Example
Constant	0	-9
Linear	1	$x - 4$
Quadratic	2	$x^2 + 3x - 1$
Cubic	3	$x^3 + 2x^2 + x + 1$
Quartic	4	$2x^4 + x^3 + 3x^2 + 4x - 1$
Quintic	5	$7x^5 + x^4 - x^3 + 3x^2 + 2x - 1$

Example 2 – Classifying Polynomials

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

A. $2x + 4x^3 - 1$

B. $7x^3 - 11x + x^5 - 2$

To add or subtract polynomials, combine like terms. You can add or subtract horizontally or vertically.

Example 3 – Adding Subtracting Polynomials

Add or subtract. Write your answer in standard form.

A. $(3x^2 + 7 + x) + (14x^3 + 2 + x^2 - x)$

B. $(1 - x^2) - (3x^2 + 2x - 5)$

A **polynomial function** is a function whose rule is a polynomial. In this course, you will study only polynomial function with one variable.

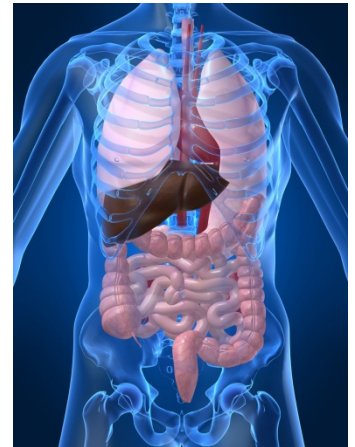
Example 4 – Medical Application

Cardiac output is the amount of blood pumped through the heart. The output is measured by a technique called dye dilution. A doctor injects dye into a vein near the heart and measures the amount of dye in the arteries over time.

The cardiac output of a particular patient can be approximated by the function $f(t) = 0.0056t^3 - 0.22t^2 + 2.33t$, where t represents time (in seconds after injection, $0 \leq t \leq 23$) and $f(t)$ represents the concentration of dye (in milligrams per liter).

- a. Evaluate $f(t)$ for $t = 0$ and $t = 3$.

- b. Describe what the values of the function from part a represent.



Graphing polynomial functions can be a challenge. Throughout this chapter, you will learn skills for analyzing, describing, and graphing higher-degree polynomials. Until then, the graphing calculator will be a useful tool.

Example 5 – Graphing Higher-Degree Polynomials on a Calculator

Graph each polynomial function on a calculator. Describe the graph, and identify the number of real zeros.

A. $f(x) = x^3 - x$

B. $3x^3 + 2x + 1$

c. $h(x) = x^4 - 8x^2 + 1$

D. $k(x) = x^4 + x^3 - x^2 + 2x - 3$