Finding Real Roots of Polynomial Equations

Who uses this?
Package designers can use roots of polynomial equations to set production specifications.

Previously, you have learned several methods for factoring polynomials. As with some quadratic equations, factoring a polynomial equation is one way to find its real roots.

Recall the Zero Product Property. You can find the roots, or solutions, of the polynomial equation $P(x) = 0$ by setting each factor equal to 0 and solving for x.

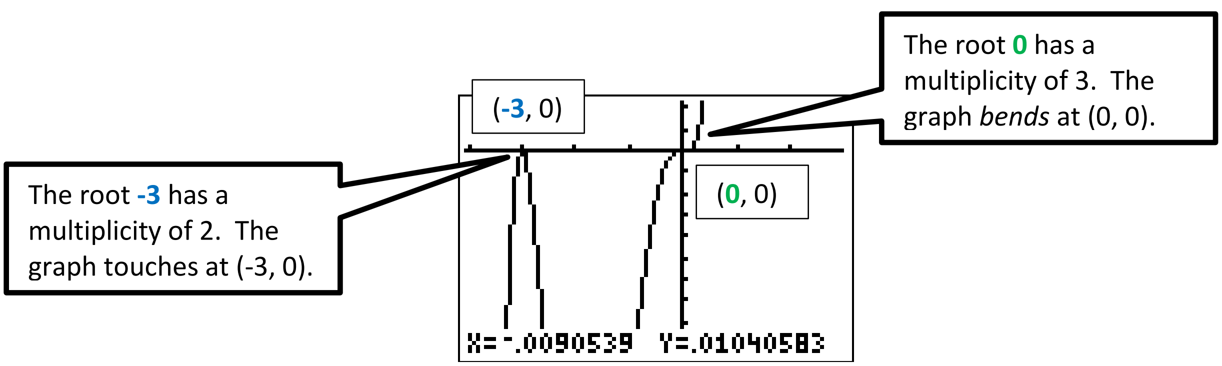
Example 1 – Using Factoring to Solve Polynomial Equations
Solve each polynomial equation by factoring.

A. $3x^5 + 18x^4 + 27x^3 = 0$

B. $x^4 - 13x^2 = -36$

Sometimes a polynomial equation has a factor that appears more than once. This creates a multiple root. In Example 1A, $3x^5 + 18x^4 + 27x^3 = 0$ has two multiple roots, 0 and -3. For example, the root 0 is a factor **three** times because $3x^3 = 0$.

The multiplicity of root r is the number of times that x – r is a factor of P(x). When a real root has even multiplicity, the graph of y = P(x) touches the x-axis but does not cross it. When a real root has odd multiplicity greater than 1, the graph "bends" as it crosses the x-axis.



You cannot always determine the multiplicity of a root a graph. It is easiest to determine multiplicity when the polynomial is in factored form.

Example 2 – Identifying Multiplicity

Identify the roots of each equation. State the multiplicity of each root.

A. $x^3 - 9x^2 + 27x - 27 = 0$

B. $-2x^3 - 12x^2 + 30x + 200 = 0$

Not all polynomials are factorable, but the Rational Root Theorem can help you find all possiable rational roots of a polynomial equation.

Rational Root Theorem
If the polynomial $P(x)$ has integer coefficients, then every rational root of the polynomial equation $P(x) = 0$ can be written in the form $\frac{p}{q}$, where p is a factor of the constant term of $P(x)$ and q is a factor of the leading coefficient of $P(x)$.

Example 3 – Marketing Application

A popcorn producer is designing a new box for the popcorn. The marketing department has designed a box with the width 2 inches less than the length and with the height 5 inches greater than the length. The volume of each box must be 24 cubic inches. What is the length of the box?

Step 1 - Write an equation to model the volume of the box.

Step 2 - Use Rational Root Theorem to identify all possible rational roots.

Step 3 - Test the possible roots to find one that is actually a root. The length must be positive, so try only positive rational roots.

Step 4 - Factor the polynomial.

Polynomial equations may also have irrational roots.

Irrational Root Theorem
If the polynomial $P(x)$ has rational coefficients and $a + b\sqrt{c}$ is a root of the polynomial equation $P(x) = 0$, where a and b are rational and $\sqrt{c}$ is irrational, then $a - b\sqrt{c}$ is also root of $P(x) = 0$.

The Irrational Root Theorem says that irrational roots of the form a + b $\sqrt{c}$ come in conjugate pairs. For example, if you know that $1 + \sqrt{2}$ is a root of $x^3 - x^2 - 3x - 1 = 0$, then you know that $1 - \sqrt{2}$ is also a root.

Recall that the real numbers are made up of the rational and the irrational numbers. You can use the Rational Root Theorem and the

Example 4 – Identifying All of the Real Roots of a Polynomial Equation
Identify all of the real roots of $4x^4 - 21x^3 + 18x^2 + 19x - 6 = 0$.

Step 1: Use the Rational Root Theorem to identify possible rational roots.

Step 2: Graph the polynomial to find the x-intercepts.

Step 3: Test the possible rational roots 2 and – ¾.

GET ORGANIZED: Give roots that satisfy each theorem and write a polynomial equation that has those roots.

Theorem	**Roots**	**Polynomial**
Rational Root Theorem		
Irrational Root Theorem		