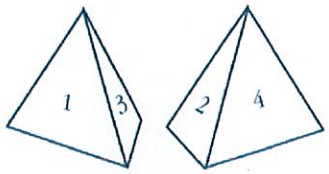
Name _____ Period _____ Date _____

Expected Value with Different Dice

Suppose you have two dice shaped like tetrahedrons. A **tetrahedron** is a solid 3-dimensional figure with four faces, as suggested in the pictures below.



One of these special dice has the numbers 1, 3, 5, and 7 on it, one number on each face. The other die has the numbers 2, 4, 6, and 8 on it, one number on each face. When one of these dice is "rolled," each face is equally likely to end up on the bottom. The number on the bottom is considered the "result" when you "roll" one of these dice.

Using this special pair of dice, you and a friend make up a game. You roll the two dice. If the product of the numbers on the bottom of the two dice is a multiple of 12, you win 24 points. If the product of the two numbers on the bottom of the two dice is a multiple of 10, your friend wins 8 points.

In order to determine if this game is fair, we need to find the expected value. In a game or probability situation, the **expected value** is the average amount gained or lost per turn in the long run. The expected value can be calculated using the theoretical probabilities and the point values. In order to find the theoretical probabilities of rolling the two dice and getting a product that is a multiple of 12 or of 10, we need to find the **sample space** (the list of all possible outcomes) in an organized way.

SAMPLE SPACE

X	1	3	5	7
2	2	6	10	14
4	4	12	20	28
6	6	18	30	42
8	8	24	40	56

Using the sample space we are able to calculate the probability of rolling the two dice and getting a product that is a multiple of 12 or of 10.

$P(\text{multiple of 12}) = \frac{2}{16}$ and $P(\text{multiple of 10}) = \frac{4}{16}$

Since the denominators for the probabilities are both 16, we can use 16 as the number of times that you and your friend played the game. If you and your friend played the game 16 times, you would win 2 times and you would score 48 points (2 wins x 24 points per win = 48 points). Since you scored a total of 48 points, your expected value would be: Expected Value = $\frac{48 \text{ total points}}{16 \text{ games}}$ = 3 points per game. Your friend would win 4 times and would score 32 points (4 wins x 8 points per win = 32 points). Since he scored a total of 32 points, his expected value would be: Expected Value = $\frac{32 \text{ total points}}{16 \text{ games}}$ = 2 points per game.

QAR: _____

1. What is a tetrahedron?

a solid 3-dimensional figure with four faces

QAR: _____

2. Define expected value.

the average amount gained or lost per turn in the long run

QAR: _____

3. Is the game you are playing with your friend a fair game? Justify your answer.

No, I have a higher expected value (3 points/game) than my friend (2 points/game)

QAR: _____

4. Suppose you and your friend change the rules of the game. Instead of finding the product of the two numbers on the bottom of the dice, you find the sum of the two numbers. If the sum is a prime number, you win 20 points. If the sum is a composite number, your friend wins 44 points. Is this a fair game? Justify your answer.

① Sample Space

+	1	3	5	7
2	(3)	(5)	(7)	9
4	(5)	(7)	9	(11)
6	(7)	9	(11)	(13)
8	9	(11)	(13)	15

② Theoretical Probability

you: $P(\text{prime}) = \frac{11}{16}$

your friend: $P(\text{composite}) = \frac{5}{16}$

③ Expected Value

16 games:

prime: 11 times × 20 points =
220 points ÷ 16 games
= 13.75 points/game

composite: 5 times × 44 points =
220 points ÷ 16 games
= 13.75 points/game

④ It is a fair game because the expected values are the same!