Applications

1. Plans for a set of stairs for the front of a new community center use the ratio of rise to run of 2 units to 5 units.
   a. Are these stairs within carpenters’ guidelines, which state that the ratio of rise to run should be between 0.45 and 0.60?
   b. Sketch a set of stairs that meets the rise-to-run ratio of 2 units to 5 units.
   c. Sketch the graph of a line where the y-values change by 2 units for each 5-unit change in the x-values.
   d. Write an equation for your line in part (c).

2. a. Find the horizontal distance and the vertical distance between the two points at the right.
   b. What is the slope of the line?
3. Seven possible descriptions of lines are listed below.
   i. positive slope
   ii. negative slope
   iii. y-intercept equals 0
   iv. passes through the point (1, 2)
   v. slope of zero
   vi. positive y-intercept
   vii. negative y-intercept

For each equation, list all of the descriptions i–vii that describe the graph of that equation.

a. \( y = 2x \)
   b. \( y = 3 - 3x \)
   c. \( y = 2x + 3 \)
   d. \( y = 5x - 3 \)
   e. \( y = 2 \)

For Exercises 4–7, find the slope and the y-intercept of the line associated with the equation.

4. \( y = 10 + 3x \)
5. \( y = 0.5x \)
6. \( y = -3x \)
7. \( y = -5x + 2 \)

In Exercises 8–12, the tables represent linear relationships. Give the slope and the y-intercept of the graph of each relationship. Then determine which of the five equations listed below fits each relationship.

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>3.5</td>
<td>4.5</td>
<td>5.5</td>
<td>6.5</td>
<td>7.5</td>
</tr>
</tbody>
</table>

| \( x \) | 1 | 2 | 3 | 4 |
|---|---|---|---|
| \( y \) | 1 | 3 | 5 | 7 |

| \( x \) | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| \( y \) | 5 | 3 | 1 | -1 | -3 |

| \( x \) | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|
| \( y \) | -11 | -14 | -17 | -20 | -23 |
13. **a.** Find the slope of the line represented by the equation 
   \[ y = x - 1. \]
   **b.** Make a table of \( x \)- and \( y \)-values for the equation \( y = x - 1 \).
   How is the slope related to the table entries?

14. **a.** Find the slope of the line represented by the equation 
   \[ y = -2x + 3. \]
   **b.** Make a table of \( x \)- and \( y \)-values for the equation \( y = -2x + 3 \).
   How is the slope related to the table entries?

15. In parts (a) and (b), the equations represent linear relationships.
   Use the given information to find the value of \( b \).
   **a.** The point \((1, 5)\) lies on the line representing \( y = b - 3.5x \).
   **b.** The point \((0, -2)\) lies on the line representing \( y = 5x - b \).
   **c.** What are the \( y \)-intercepts in the linear relationships in parts (a) and (b)?
   What are the patterns of change for the linear relationships in parts (a) and (b)?
   **d.** Find the \( x \)-intercepts for the linear relationships in parts (a) and (b).
   (The \( x \)-intercept is the point where the graph intersects the \( x \)-axis.)

For each pair of points in Exercises 16–19, do parts (a)–(e).
   **a.** Plot the points on a coordinate grid and draw a line through them.
   **b.** Find the slope of the line.
   **c.** Find the \( y \)-intercept from the graph. Explain how you found the \( y \)-intercept.
   **d.** Use your answers from parts (b) and (c) to write an equation for the line.
   **e.** Find one more point that lies on the line.

16. \((0, 0)\) and \((3, 3)\)  
17. \((-1, 1)\) and \((3, -3)\)  
18. \((0, -5)\) and \((-2, -3)\)  
19. \((3, 6)\) and \((5, 6)\)
For Exercises 20–22, determine which of the linear relationships A–K fit each description.

20. The line representing this relationship has positive slope.
21. The line representing this relationship has a slope of –2.
22. The line representing this relationship has a slope of 0.

23. Decide which graph from Exercises 20–22 matches each equation.
   a. \( y = x - 1 \)  
   b. \( y = -2 \)  
   c. \( y = \frac{1}{4}x \)
For each equation in Exercises 24–26, do parts (a)–(d).

24. \( y = x \)  
25. \( y = 2x - 2 \)  
26. \( y = -0.5x + 2 \)

a. Make a table of \( x \)- and \( y \)-values for the equation.  
b. Sketch a graph of the equation.  
c. Find the slope of the line.  
d. Make up a problem that can be represented by each equation.

27. a. Graph a line with slope 3.
   i. Find two points on your line.  
   ii. Write an equation for the line.  

b. On the same set of axes, graph a line with slope \(-\frac{1}{3}\).  
   i. Find two points on your line.  
   ii. Write an equation for the line.  

c. Compare the two graphs you made in parts (a) and (b).

28. Use the line in the graph below to answer each question.

a. Find the equation for a line that is parallel to this line.  
b. Find the equation of a line that is perpendicular to this line.

29. Descriptions of three possible lines are listed below.
   • a line that does not pass through the first quadrant  
   • a line that passes through exactly two quadrants  
   • a line that passes through only one quadrant  

a. For each, decide whether such a line exists. Explain.  
b. If a line exists, what must be true about the equation of the line that satisfies the conditions?  
c. Sketch a graph, then write the equation of the line next to the graph.
30. a. Find the slope of each line. Then, write an equation for the line.

i.

ii.

iii.

b. Compare the slopes of the three lines.

b. How are the three graphs similar? How are they different?
31. The slopes of two lines are the negative reciprocal of each other. For example:

\[ y = 2x \quad \text{and} \quad y = -\frac{1}{2}x \]

What must be true about the two lines? Is your conjecture true if the \( y \)-intercept of either equation is not zero? Explain.

32. At noon, the temperature is 30°F. For the next several hours, the temperature falls by an average of 3°F an hour.
   a. Write an equation for the temperature \( T \), \( n \) hours after noon.
   b. What is the \( y \)-intercept of the line the equation represents? What does the \( y \)-intercept tell us about this situation?
   c. What is the slope of the line the equation represents? What does the slope tell us about this situation?

33. Natasha never manages to make her allowance last for a whole week, so she borrows money from her sister. Suppose Natasha borrows 50 cents every week.
   a. Write an equation for the amount of money \( m \) Natasha owes her sister after \( n \) weeks.
   b. What is the slope of the graph of the equation from part (a)?

34. In 1990, the small town of Cactusville was destined for obscurity. However, due to hard work by its city officials, it began adding manufacturing jobs at a fast rate. As a result, the city’s population grew 239% from 1990 to 2000. The population of Cactusville in 2000 was 37,000.
   a. What was the population of Cactusville in 1990?
   b. Suppose the same rate of population increase continues. What might the population be in the year 2010?
35. James and Shani share a veterinary practice. They each make farm visits two days a week. They take cellular phones on these trips to keep in touch with the office. James makes his farm visits on weekdays. His cellular phone rate is $14.95 a month plus $0.50 a minute. Shani makes her visits on Saturday and Sunday and is charged a weekend rate of $34 a month.
   a. Write an equation for each billing plan.
   b. Is it possible for James’s cellular phone bill to be more than Shani’s? Explain how you know this.
   c. Suppose James and Shani made the same number of calls in the month of May. Is it possible for James’s and Shani’s phone bills to be for the same amount? If so, how many minutes of phone calls would each person have to make for their bills to be equal?
   d. Shani finds another phone company that offers one rate for both weekday and weekend calls. The billing plan for this company can be expressed by the equation $A = 25 + 0.25m$, where $A$ is the total monthly bill and $m$ is the number of minutes of calls. Compare this billing plan with the other two plans.

Connections

36. In Europe, many hills have signs indicating their steepness, or slope. Two examples are shown at the right.
   On a coordinate grid, sketch hills with each of these slopes.

37. Solve each equation and check your answers.
   a. $2x + 3 = 9$
   b. $\frac{1}{2}x + 3 = 9$
   c. $x + 3 = \frac{9}{2}$
   d. $x + \frac{1}{2} = 9$
   e. $\frac{x + 3}{2} = 9$

38. Use properties of equality and numbers to solve each equation for $x$. Check your answers.
   a. $3 + 6x = 4x + 9$
   b. $6x + 3 = 4x + 9$
   c. $6x - 3 = 4x + 9$
   d. $3 - 6x = 4x + 9$
39. Use the graph to answer each question.

![Graph with rectangles]

a. Are any of the rectangles in the picture above similar? If so, which rectangles, and explain why they are similar.

b. Find the slope of the diagonal line. How is it related to the similar rectangles?

c. Which of these rectangles belong to the set of rectangles in the graph? Explain.

40. The graph below shows the height of a rocket from 10 seconds before liftoff through 7 seconds after liftoff.

a. Describe the relationship between the height of the rocket and time.

b. What is the slope for the part of the graph that is a straight line? What does this slope represent in this situation?
41. Solve each equation. Check your answers.
   a. \(2(x + 5) = 18\)  
   b. \(2(x + 5) = x - 8\)  
   c. \(2(x + 5) = x\)  
   d. \(2(x + 5) = -15\)

42. **Multiple Choice** Which equation has a graph that contains the point \((-1, 6)\)?
   A. \(y = 4x + 1\)  
   B. \(y = -x + 5\)  
   C. \(y = 3x - 11\)  
   D. \(y = -3x + 11\)

43. Each pair of figures is similar. Find the lengths of the sides marked \(x\).

44. Find a value of \(n\) that will make each statement true.
   a. \(\frac{n}{10} = \frac{3}{2}\)  
   b. \(\frac{5}{6} = \frac{n}{18}\)  
   c. \(-\frac{4}{6} = \frac{n}{3}\)  
   d. \(\frac{5}{18} = \frac{20}{n}\)

   e. Write an equation for a line whose slope is \(-\frac{4}{6}\).

45. Find a value of \(n\) that will make each statement true.
   a. \(15\% (90) = n\)  
   b. \(20\% (n) = 80\)  
   c. \(n\% (50) = 5\)
**Extensions**

46. On a March flight from Boston to Detroit, a monitor displayed the altitude and the outside air temperature. Two passengers that were on that flight tried to find a formula for temperature $t$ in degrees Fahrenheit at an altitude of $a$ feet above sea level. One passenger said the formula was $t = 46 - 0.003a$, and the other said it was $t = 46 + 0.003a$.

   **a.** Which formula makes more sense to you? Why?
   **b.** The Detroit Metropolitan Airport is 620 feet above sea level. Use the formula you chose in part (a) to find the temperature at the airport on that day.
   **c.** Does the temperature you found in part (b) seem reasonable? Why or why not?

47. Andy's track team decides to convert their running rates from miles per hour to kilometers per hour (1 mile $< 1.6$ kilometers).

   **a.** Which method would you use to help the team do their converting: graph, table, or equation? Explain why you chose your method.
   **b.** One of Andy's teammates said that he could write an equation for his spreadsheet program that could convert any team member's running rate from miles per hour to kilometers per hour. Write an equation that each member could use for this conversion.