Applications

1. Figures A–F are parallelograms.

   a. List all the pairs of similar parallelograms.
   b. For each pair of similar parallelograms, find the ratio of two adjacent side lengths in one parallelogram and compare it to the ratio of the corresponding side lengths in the other parallelogram.
   c. For each pair of similar parallelograms, find the scale factor from one shape to the other. Explain how the information given by the scale factors is different from the information given by the ratios of side lengths.

2. For parts (a)–(c), use the triangles below and on the next page.

   Triangle A
   - 6.5 in.
   - 25°
   - 3 in.
   - 136°
   - 4 in.

   Triangle B
   - 1.5 in.
   - 25°
   - 2 in.
   - 3.25 in.
   - 19°
Investigation 4  Similarity and Ratios

**a.** List all the pairs of similar triangles.

**b.** For each pair of similar triangles, find the ratio of two side lengths in one triangle and the ratio of the corresponding pair of side lengths in the other. How do these ratios compare?

**c.** For each pair of similar triangles, find the scale factor from one shape to the other. Explain how the information given by the scale factors is different than the information given by the ratios of side lengths.

**3. a.** On grid paper, draw two similar rectangles so that the scale factor from one rectangle to the other is 2.5. Label the length and width of each rectangle.

**b.** Find the ratio of the length to the width for each rectangle.

**4. a.** Draw a third rectangle that is similar to one of the rectangles in Exercise 3. Find the scale factor from one rectangle to the other.

**b.** Find the ratio of the length to the width for the new rectangle.

**c.** What can you say about the ratios of the length to the width for the three rectangles? Is this true for another rectangle that is similar to one of the three rectangles? Explain.
For Exercises 5–8, each pair of figures is similar. Find the missing measurement. (Note: Although each pair of figures is drawn to scale, the scales for Exercises 5–8 are not the same.)

5. \[ \text{4 cm} \quad \text{5 cm} \]
   \[ \text{3 cm} \quad \text{2 cm} \]
6. \[ \text{8.75 cm} \quad \text{10.5 cm} \]
   \[ \text{7 cm} \quad \text{2.5 cm} \]
7. \[ \text{3 cm} \quad \text{3 cm} \]
   \[ \text{60°} \quad \text{4 cm} \]
   \[ \text{3 cm} \quad \text{4 cm} \]
8. \[ \text{10 cm} \quad \text{3 cm} \]
   \[ \text{5 cm} \quad \text{4 cm} \]

For Exercises 9–11, rectangles A and B are similar.

9. **Multiple Choice** What is the value of \( x \)?
   
   A. 4  
   B. 12  
   C. 15  
   D. \( 33\frac{1}{3} \)

10. What is the scale factor from rectangle B to rectangle A?

11. Find the area of each rectangle. How are the areas related?
12. Rectangles C and D are similar.

\[ \begin{array}{ccc} & \text{C} & 4 \text{ in.} \\
8 \text{ in.} & x & 1 \text{ in.} \\
\end{array} \]

a. What is the value of \( x \)?

b. What is the scale factor from rectangle C to rectangle D?

c. Find the area of each rectangle. How are the areas related?

13. Suppose you want to buy new carpeting for your bedroom. The bedroom floor is a 9-foot-by-12-foot rectangle. Carpeting is sold by the square yard.

a. How much carpeting do you need to buy?

b. The carpeting costs $22 per square yard. How much will the carpet for the bedroom cost?

14. Suppose you want to buy the same carpet described in Exercise 13 for a library. The library floor is similar to the floor of the 9-foot-by-12-foot bedroom. The scale factor from the bedroom to the library is 2.5.

a. What are the dimensions of the library? Explain.

b. How much carpeting do you need for the library?

c. How much will the carpet for the library cost?
Connections

For Exercises 15–20, tell whether each pair of ratios is equivalent.

15. 3 to 2 and 5 to 4  
16. 8 to 4 and 12 to 8  
17. 7 to 5 and 21 to 15  
18. 1.5 to 0.5 and 6 to 2  
19. 1 to 2 and 3.5 to 6  
20. 2 to 3 and 4 to 6

21. Choose a pair of equivalent ratios from Exercises 15–20. Write a similarity problem that uses the ratios. Explain how to solve your problem.

For Exercises 22–25, write two other ratios equivalent to the given ratio.

22. 5 to 3  
23. 4 to 1  
24. 3 to 7  
25. 1.5 to 1

26. Here is a picture of Duke, a real dog. The scale factor from Duke to the picture is 12.5%. Use an inch ruler to make any measurements.

a. How long is Duke from his nose to the tip of his tail?

b. To build a doghouse for Duke, you need to know his height so you can make a doorway to accommodate him. How tall is Duke?

c. The local copy center has a machine that prints on poster-size paper. You can enlarge or reduce a document with a setting between 50% and 200%. How can you use the machine to make a life-size picture of Duke?
27. Samantha draws triangle $ABC$ on a grid. She applies a rule to make the triangle on the right.

a. What rule did Samantha apply to make the new triangle?

b. Is the new triangle similar to triangle $ABC$? Explain. If the triangles are similar, give the scale factor from triangle $ABC$ to the new triangle.

28. a. Find the ratio of the circumference to the diameter for each circle.

b. How do the ratios you found in part (a) compare? Explain.

For Exercises 29–30, read the paragraph below.

The Rosavilla School District wants to build a new middle school building. They ask architects to make scale drawings of possible layouts for the building. The district narrows the possibilities to the layouts shown.

29. a. Suppose the layouts above are on centimeter grid paper. What is the area of each scale drawing?

b. What will be the area of each building?

30. Multiple Choice The board likes the L-shaped layout but wants a building with more space. They increase the L-shaped layout by a scale factor of 2. For the new layout, choose the correct statement.

A. The area is two times the original.
B. The area is four times the original.
C. The area is eight times the original.
D. None of the statements above is correct.
31. Use the table for parts (a)–(c).

**Student Heights and Arm Spans**

<table>
<thead>
<tr>
<th>Height (in.)</th>
<th>60</th>
<th>65</th>
<th>63</th>
<th>50</th>
<th>58</th>
<th>66</th>
<th>60</th>
<th>63</th>
<th>67</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm Span (in.)</td>
<td>55</td>
<td>60</td>
<td>60</td>
<td>48</td>
<td>60</td>
<td>65</td>
<td>60</td>
<td>67</td>
<td>62</td>
<td>70</td>
</tr>
</tbody>
</table>

a. Find the ratio of arm span to height for each student. Write the ratio as a fraction. Then write the ratio as an equivalent decimal. How do the ratios compare?

b. Find the mean of the ratios.

c. Use your answer from part (b). Predict the arm span of a person who is 62 inches tall. Explain.

32. Suppose you enlarge this spinner by a factor of 3. Does this change the probabilities of the pointer landing in any of the areas? Explain.

33. Suppose you enlarge the square dartboard below by a scale factor of 3. Will the probabilities that the dart will land in each region change? Explain.
34. For each angle measure, find the measure of its complement and the measure of its supplement.

**Sample** 30°
- complement: 60°
- supplement: 150°

**a.** 20°  **b.** 70°  **c.** 45°

**Extensions**

35. For parts (a)–(e), use the similar triangles below.

![Similar Triangles Diagram]

**a.** What is the scale factor from the smaller triangle to the larger triangle? Give your answer as a fraction and a decimal.

**b.** Choose any side of the larger triangle. What is the ratio of the length of this side to the corresponding side length in the smaller triangle? Write your answer as a fraction and as a decimal. How does the ratio compare to the scale factor in part (a)?

**c.** What is the scale factor from the larger triangle to the smaller triangle? Write your answer as a fraction and a decimal.

**d.** Choose any side of the smaller triangle. What is the ratio of the length of this side to the corresponding side length in the larger triangle? Write your answer as a fraction and as a decimal. How does the ratio compare to the scale factor in part (c)?

**e.** Is the pattern for scale factors and ratios in this exercise the same for any pair of similar figures? Explain.
36. For parts (a) and (b), use a straightedge and an angle ruler or protractor.
   a. Draw two different triangles that each have angle measures of 30°, 60°, and 90°. Do the triangles appear to be similar?
   b. Draw two different triangles that each have angle measures of 40°, 80°, and 60°. Do the triangles appear to be similar?
   c. Based on your findings for parts (a) and (b), make a conjecture about triangles with congruent angle measures.

37. Which rectangle below do you think is “most pleasing to the eye?”

The question of what shapes are attractive has interested builders, artists, and craftspeople for thousands of years. The ancient Greeks were particularly attracted to rectangular shapes similar to rectangle B above. They referred to such shapes as “golden rectangles.” They used golden rectangles frequently in buildings and monuments.

The photograph of the Parthenon (a temple in Athens, Greece) below shows several examples of golden rectangles.
The ratio of the length to the width in a golden rectangle is called the “golden ratio.”

**a.** Measure the length and width of rectangles A, B, and C in inches. In each case, estimate the ratio of the length to the width as accurately as possible. The ratio for rectangle B is an approximation of the golden ratio.

**b.** Measure the dimensions of the three golden rectangles in the photograph in centimeters. Write the ratio of length to width in each case. Write each ratio as a fraction and then as a decimal. Compare the ratios to each other and to the ratio for rectangle B.

**c.** You can divide a golden rectangle into a square and a smaller rectangle similar to the original rectangle.

Copy rectangle B from the previous page. Divide this golden rectangle into a square and a rectangle. Is the smaller rectangle a golden rectangle? Explain.
38. For parts (a) and (b), use the triangles below.

a. Identify the triangles that are similar to each other. Explain.

b. For each triangle, find the ratio of the base to the height. How do these ratios compare for the similar triangles? How do these ratios compare for the non-similar triangles?

39. The following sequence of numbers is called the Fibonacci sequence. It is named after an Italian mathematician in the 14th century who contributed to the early development of algebra.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377 . . .

a. Look for patterns in this sequence. Figure out how the numbers are found. Use your idea to find the next four terms.

b. Find the ratio of each term to the term before. For example, 1 to 1, 2 to 1, 3 to 2, and so on. Write each of the ratios as a fraction and then as an equivalent decimal. Compare the results to the golden ratios you found in Exercise 37. Describe similarities and differences.