Graphing Exponential Functions

Graph each equation using a table of values. Include $x$ values which are both negative and positive. Graph the equations on a separate piece of graph paper.

Example: $y = 2^x$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1/2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

1. $y = 3^x$

| $x$ | $y$ |

2. $y = 10 \cdot 3^x$

| $x$ | $y$ |

3. $y = 10 \cdot 2^x$

| $x$ | $y$ |

4. $y = \left(\frac{1}{2}\right)^x$

| $x$ | $y$ |

5. $y = \left(\frac{1}{5}\right)^x$

| $x$ | $y$ |

6. $y = 50 \cdot 4^x$

| $x$ | $y$ |

7. $y = 10^x$

| $x$ | $y$ |

8. $y = \left(\frac{1}{10}\right)^x$

| $x$ | $y$ |

9. $y = 50 \cdot 2^x$

| $x$ | $y$ |

What do each of these sets of functions have in common?

Problems 1, 4, 5, 7, and 8:
Problems 4, 7, and 8:
Calculating Compound Interest

Compound Interest

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

where \( A \) = amount, \( P \) = principal, \( r \) = rate, \( t \) = time in years, and \( n \) = number of times compounded per year.

Solve the story problems assuming no deposits or withdrawals.

1. Heather received $100 for her 13th birthday. If she saves it in a bank with 3% interest compounded quarterly, how much money will she have in the bank by her 16th birthday?

2. Roland earned $1,500 last summer. If he deposited the money in a certificate of deposit that earns 4% interest compounded monthly, how much money will he have next summer?

3. The C.R.E.A.M. Company has an employee savings plan. If an employee makes an initial contribution of $2,500 and the company pays 5% interest compounded quarterly, how much money will the employee have after 10 years?

4. Juan invests $7,500 at 6% interest for one year. How much money would he have if the interest were compounded
   a. Yearly?
   b. Daily?
   c. Why are the amounts in answers a and b different?

5. Carmen is saving for a new car that costs $15,000. If she puts $5,000 in an account that earns 6% interest compounded monthly, how long will it take for her to save enough money to buy the car?
Exponential Decay (Half-life)

\[ y = a \left( \frac{1}{2} \right)^x \]
where
- \( a \) = initial amount
- \( x \) = number of half-lives \( = \frac{\text{time}}{\text{half-life}} \)
- \( y \) = remaining

Solve each problem.

1. There are 10 grams of Curium-245 which has a half-life of 9,300 years. How many grams will remain after 37,200 years?

2. There are 80 grams of Cobalt-58 which have a half-life of 71 days. How many grams will remain after 213 days?

3. The half-life of Rhodium-105 is 1.5 days. If there are initially 7500 grams of this isotope, how many grams would remain after 30 days?

4. Two hundred ten years ago there were 132,000 grams of Cesium-137. How much is there today? The half-life of Cesium is 30 years.

5. In a nuclear reaction, 150 grams of Plutonium-239 are produced. How many grams would remain after one million years? The half-life of Plutonium-239 is 24,400 years.

6. Using carbon dating, scientists can determine how old a fossil is by how much Carbon-14 is present. If an average animal carcass contains 1 gram of Carbon-14, how old is a fossil with 0.0625 grams of Carbon-14? The half-life of Carbon-14 is 5730 years.
Manipulating Common Logs (Base 10)

\[ y = \log_b x \quad \text{where } b = \text{base} \]

Common logarithm \( b = 10 \).

When no base is given, assume base 10.

\[ y = \log_{10} x \text{ is equivalent to } 10^y = x \]

Solve without using a calculator.

**Example:** \( \log_{10} 100 = y \)
\[
10^y = 100 \\
y = 2
\]

1. \( \log 1000 \)

2. \( \log \sqrt[10]{10} \)

3. \( \log \sqrt[10]{10^2} \)

4. \( \log 0.1 \)

5. \( \log 0.0001 \)

6. \( \log \sqrt[10]{10} \)

7. \( \log \sqrt[10]{10} \)

8. \( \log 10^6 \)

9. \( \log 1 \)

10. \( \log 10,000 \)
Converting from Logarithmic to Exponential Form

Convert each equation from logarithmic form to exponential form or from exponential to logarithmic. \( y = \log_b x \leftrightarrow b^y = x \)

**Example:** \( \log_{11} 121 = 2 \) \( 11^2 = 121 \)

1. \( 5^3 = 125 \)

2. \( 10^6 = 1,000,000 \)

3. \( \log_{10} 1 = 0 \)

4. \( \log_3 \frac{1}{243} = -5 \)

5. \( 7^5 = 16,807 \)

6. \( y = \log x \)

7. \( 12^x = 87 \)

8. \( y = \log_{15} 30 \)

9. \( y = \log_5 x \)

10. \( y = \log_{180} B \)

11. \( 10^y = x \)

12. \( \log_b 64 = 3 \)

13. \( \log_x 5 = 10 \)

14. \( 7^x = 343 \)
Graphing Logarithms

Complete the table of values and graph each function on graph paper.

Example:

$$y = \log_{10} x$$

- Convert to exponential form.
  $$10^y = x$$
- Choose $y$ values that are both positive and negative.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{100}$</td>
<td>-2</td>
</tr>
<tr>
<td>$\frac{1}{10}$</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>100</td>
<td>2</td>
</tr>
<tr>
<td>1000</td>
<td>3</td>
</tr>
</tbody>
</table>

1. $y = \log_3 x$

2. $y = \log_5 x$

3. $y = \log_7 x$

4. $y = \log_2 x$

5. $y = \log_4 x$

6. $y = \log_{11} x$

7. $y = \log_{15} x$
Simplifying Logarithms

product property: \( \log_b (m \cdot n) = \log_b m + \log_b n \)

quotient property: \( \log_b \left( \frac{m}{n} \right) = \log_b m - \log_b n \)

power property: \( \log_b (m^n) = n \cdot \log_b m \)

Simplify.

**Example 1:** \( \log_5 6 + \log_5 8 = \log_5 (6 \cdot 8) = \log_5 48 \)

**Example 2:** \( \log_7 9 - \log_7 3 = \log_7 \frac{9}{3} = \log_7 3 \)

**Example 3:** \( \log_{12} 6^3 = 3 \log_{12} 6 \)

1. \( \log_9 4 + \log_9 6 \)
2. \( \log_{12} 12 + \log_{12} 11 \)
3. \( \log_{16} 36 - \log_{16} 12 \)
4. \( \log 3 - \log 2 \)
5. \( \log 14^6 \)
6. \( \log_{20} 10^{16} \)
7. \( \log_3 16 + \log_2 4 \)
8. \( \log 10 + \log 10 \)
9. \( \log 125 \)
10. \( \log_2 2^4 \)
Simplifying and Solving Logarithms

Simplify each expression, then solve. Place the letter of the correct answer above the problem number below.

**Example 1:** \( \log_3 x - \log_3 4 = \log_3 12 \)
\[ \log_3 \left( \frac{x}{4} \right) = \log_3 (12) \]
therefore \( \frac{x}{4} = 12 \)
\( x = 48 \)

**Example 2:** \( \log_5 7 + \frac{1}{2} \log_5 4 = \log_5 x \)
\[ \log_5 7 + \log_5 4^\frac{1}{2} = \log_5 x \]
\[ \log_5 14 = \log_5 x \]
\( x = 14 \)

1. \( \log_3 x - 2 \log_3 2 = 3 \log_3 3 \)
   - M. 23
   - N. 108
   - O. 6^2

2. \( \log_2 x = 9 \)
   - A. 18
   - E. 512
   - I. 81

3. \( \log_2 128 = x \)
   - C. 16
   - D. 64
   - E. 7

4. \( \log_x 144 = -2 \)
   - N. 12
   - O. 72
   - P. 12

5. \( \log_2 x = \frac{1}{3} \log_2 27 \)
   - N. 3
   - O. 9
   - P. 27

6. \( \log_{16} 32 - \log_{16} 2 = x \)
   - W. 2
   - X. 1
   - Y. 16

7. \( 5 \log 2 = \log x \)
   - E. 10
   - I. 16
   - O. 32

8. \( \log_2 x - \log_2 5 = \log_2 10 \)
   - R. 25
   - S. 15
   - T. 50

A logarithm is an \( \frac{2}{6} \frac{4}{7} \frac{5}{3} \frac{1}{8} \).
Using Logarithms to Solve $B^x = A$

Solve for $x$, rounding to the nearest tenth.

Example: $5^x = 30$

1. $9^x = 27$

   $\log 5^x = \log 30$

   $x \cdot \log 5 = \log 30$

   $x = \frac{\log 30}{\log 5} = 2.1113 = 2.1$

2. $7^x = 343$

3. $10^x = 0$

4. $6^x = 127$

5. $12^x = 303$

6. $13^x = 2839$

7. $2^x = 90$

8. $4^x = 512$

9. $3^x = 5.2$

10. $11^x = 153$
28. The Pattern Game

"OK," said Ms. Wagner, "I'd like to show you a new game. I will put some numbers on this chart. Your job is to take the top number and see if you can find out how I got the number below it."

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>5</td>
<td></td>
<td>11</td>
<td></td>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

"You mean we have to find the rule?" asked Brian.
"This reminds me of a function machine," added Beth.
"You're both right. Now think carefully about how a function machine works. It takes a number and changes it into another number with the help of a rule," she said.
"Great," moaned Marco, "I can never get these."
"I love this game," answered Maggie. "We played it a lot last year. I always try to see if I can see a pattern because sometimes the rule is hard to find."
"Remember, you try to figure out how I got from the top number to the one below it. If you think you know, come up and fill in one of the blank spaces," said Ms. Wagner.

The class members filled in the rest of the chart like this:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>15</td>
</tr>
</tbody>
</table>

While discussing their strategies, they found at least three different ways to fill in the empty boxes.
"I just added each number on top to the one that comes after it. That gave me the one on the bottom," said Aaron.
"I just filled in the odd numbers on the bottom," added Maggie.
Marco said, "Wait! I think you can double the top number and add one to get the bottom number."
"Good!" said Ms. Wagner, "now I'm going to give you another chart. This time I'll mix up the numbers on the top. If you get stuck and can't figure out the bottom numbers, rewrite the chart with the top numbers in numerical order. That way you might find it easier to see the pattern. Remember that each number on top is matched with the one below it. If you move the top number to a new location, you must also move the bottom one. So give this one a try."

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>0</th>
<th>5</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

123
The Pattern Game
Problem Set A

Directions: First read the story "The Pattern Game." Information from the story and the pattern games will help you to solve some of the problems that follow. Write your answers below each problem. Where possible, use the back of this page or a separate sheet of paper to show your work.

1. Complete the last pattern game that Ms. Wagner gave. Remember to use the hint given by Ms. Wagner if you get stuck.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>7</th>
<th>3</th>
<th>0</th>
<th>5</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

2. Describe in words what you did to complete the table above. Discuss your ideas with your group or neighbor.

3. Complete this pattern game:

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>0</th>
<th>6</th>
<th>3</th>
<th>5</th>
<th>1</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>25</td>
<td></td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

Describe in words how you completed the table above. Share your ideas with your group or neighbor. Did you learn of any methods different from yours? Explain.

4. Make up a number pattern game like the one in the story. Give it to your group or neighbor to complete. Discuss the solution when finished.
The Pattern Game
Problem Set B

Directions: First read the story "The Pattern Game." Information from the story and the pattern games will help you to solve some of the problems that follow. Write your answers below each problem. Where possible, use the back of this page or a separate sheet of paper to show your work.

1. Complete the last pattern game that Ms. Wagner gave. Remember to use the hint given by Ms. Wagner if you get stuck. Then describe in words what you did to complete the table. Discuss your ideas with your group or neighbor.

<table>
<thead>
<tr>
<th>1</th>
<th>7</th>
<th>3</th>
<th>0</th>
<th>5</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

2. Complete the pattern game below.

<table>
<thead>
<tr>
<th>1</th>
<th>5</th>
<th>3</th>
<th>6</th>
<th>0</th>
<th>2</th>
<th>7</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15</td>
<td></td>
<td></td>
<td>6</td>
<td></td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>

Describe in words what you did to complete the table. Discuss your ideas with your group or neighbor.

3. Complete the pattern game below.

<table>
<thead>
<tr>
<th>12</th>
<th>4</th>
<th>6</th>
<th>2</th>
<th>5</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Describe in words what you did to complete the table. Discuss your ideas with your group or neighbor.

4. Make up a pattern game like the one in the story. Give it to your group or neighbor to complete. Discuss the solution when finished.
The Pattern Game
Problem Set C

Directions: First read the story "The Pattern Game." Information from the story and the pattern games will help you to solve some of the problems that follow. Write your answers below each problem. Where possible, use the back of this page or a separate sheet of paper to show your work.

1. Complete the last pattern game that Ms. Wagner gave. Remember to use the hint given by Ms. Wagner if you get stuck. When finished, describe in words what you did to complete the table. Discuss your ideas with your group or neighbor.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>7</th>
<th>3</th>
<th>0</th>
<th>5</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

2. Complete the pattern game below:

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>5</th>
<th>8</th>
<th>0</th>
<th>1</th>
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<th>7</th>
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</tr>
</thead>
<tbody>
<tr>
<td>6</td>
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<td></td>
<td></td>
<td></td>
<td>14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Describe in words what you did to complete the table. Discuss your ideas with your group or neighbor. Did you learn of any methods different from the one you used? Explain.

3. Complete the pattern game below:

<table>
<thead>
<tr>
<th>9</th>
<th>2</th>
<th>7</th>
<th>15</th>
<th>0</th>
<th>5</th>
<th>12</th>
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</thead>
<tbody>
<tr>
<td>6</td>
<td></td>
<td>19</td>
<td></td>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Describe in words what you did to complete the table. Discuss your ideas with your group or neighbor. Did you learn of any methods different from the one you used? Explain.

4. Make up a pattern game like the one in the story. Give it to your group or neighbor to complete. Discuss the solution when finished.