

Practice:

Determine whether the given binomial is a factor of the polynomial $P(x)$.

1. $(x + 2)$; $P(x) = 4x^2 - 2x + 5$

$$P(-2)$$

$$\begin{array}{r} -2 \\ \hline 4 & -2 & 5 \\ & \downarrow & -8 & 20 \\ & 4 & -10 & \boxed{25} \end{array}$$

NO

2. $(3x - 6)$; $P(x) = 3x^4 - 6x^2 + 3x - 30$

$$\begin{array}{r} 3x - 6 = 0 \\ +6 +6 \\ \hline 3x = 6 \\ \frac{3x}{3} = \frac{6}{3} \\ x = 2 \end{array}$$

$$\begin{array}{r} 2 \\ \hline 3 & 0 & -6 & 3 & -30 \\ & & \downarrow & & \\ & 6 & 12 & 12 & \cancel{30} \\ \hline 3 & 6 & 6 & \cancel{15} & 0 \end{array}$$

YES

Factoring each expression

3. $x^3 - 2x^2 - 9x + 18$

$$(x^3 - 2x^2) + (-9x + 18)$$

$$x^2(x - 2) - 9(x - 2)$$

$$(x - 2)(x^2 - 9) \\ (x + 3)(x - 3)$$

Factor each expression.

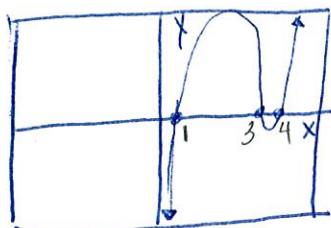
5. $8 + z^6$

$$\frac{8 + (z^2)^3}{2^3 + (z^2)^3} \\ [(2 + z^2)(2^2 - 2 \cdot z^2 + (z^2)^2)] \\ (2 + z^2)(4 - 2z^2 + z^4)$$

6. $2x^5 - 16x^2$

$$2x^2(x^3 - 8) \\ 2x^2(x^3 - 2^3) \\ 2x^2(x - 2)(x^2 + x \cdot 2 + 2^2) \\ 2x^2(x - 2)(x^2 + 2x + 4)$$

7. The volume of a rectangular prism is modeled by the function $V(x) = x^3 - 8x^2 + 19x - 12$, which is graphed at right. Identify the values of x for which $V(x) = 0$, and use the graph to factor $V(x)$.



$$x = 1, 3, 4$$

$$(x^3 - 8x^2) + (19x - 12)$$

$$x^2(x - 8) + \text{_____}$$

$$V(x) = (x - 1)(x - 3)(x - 4)$$

Name: Cajudoy
 Block: All
 Date: 11/28 - 11/29

Practice Factoring Polynomial – after test...

Determine whether the given binomial is a factor of the polynomial $P(x)$.

1. $(x - 3)$; $P(x) = 4x^6 - 12x^5 + 2x^3 - 6x^2 - 5x + 10$

$$\begin{array}{r} 3 \mid 4 - 12 \ 0 \ 2 \ -6 \ -5 \ 10 \\ \downarrow \quad 12 \ 0 \ 0 \quad 6 \ 0 \ -15 \\ \hline 4 \quad 0 \ 0 \ 2 \ 0 \ -5 \ 15 \end{array}$$

NO it is
not a factor

2. $(3x + 12)$; $P(x) = 3x^4 + 12x^3 + 6x + 24$

$$\begin{array}{r} 3x + 12 = 0 \\ -12 \quad -12 \\ \hline 3x = -12 \\ \hline 3 \quad 3 \end{array}$$

$X = -4$

$$\begin{array}{r} -4 \mid 3 \ 12 \ 0 \ 6 \ 24 \\ \downarrow \quad -12 \ 0 \ 0 \ -24 \\ \hline 3 \ 0 \ 0 \ 6 \ 24 \end{array}$$

YES it
is a
factor

Factor each expression

3. $4b^3 + 3b^2 - 16b - 12$

$$(4b^3 + 3b^2) + (-16b - 12)$$

$$b^2(4b + 3) - 4(4b + 3)$$

$$\begin{array}{r} (4b+3)(b^2-4) \\ (4b+3)(b+2)(b-2) \end{array}$$

4. $3x^3 + x^2 - 27x - 9$

$$(3x^3 + x^2) + (-27x - 9)$$

$$x^2(3x + 1) - 9(3x + 1)$$

$$(3x + 1)(x^2 - 9)$$

$$(3x + 1)(x - 3)(x + 3)$$

5. $5x^3 - x^2 - 20x + 4$

$$(5x^3 - x^2) + (-20x + 4)$$

$$x^2(5x - 1) - 4(5x - 1)$$

$$(5x - 1)(x^2 - 4) \Rightarrow (5x - 1)(x + 2)(x - 2)$$

6. $s^6 - 1$

$$(s^2)^3 - 1^3$$

$$(s^2 - 1)((s^2)^2 + s^2 \cdot 1 + 1^2)$$

$$(s^2 - 1)(s^4 + s^2 + 1)$$

7. $6x^4 - 162x$

$$6x(x^3 - 27)$$

$$6x(x^3 - 3^3)$$

$$6x(x - 3)(x^2 + x \cdot 3 + 3^2)$$

$$6x(x - 3)(x^2 + 3x + 9)$$

8. $y^5 + 27y^2$

$$y^2(y^3 + 27)$$

$$y^2(y^3 + 3^3)$$

$$y^2(y + 3)(y^2 - y \cdot 3 + 3^2)$$

$$y^2(y + 3)(y^2 - 3y + 9)$$

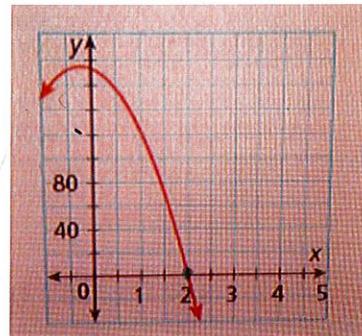
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9. **Recreation** – The volume of a bowling ball can be modeled by the function $V(x) = 168 - 28x - 28x^2$, where x represents the radius of the finger holes in inches. Identify the values of x for which $V(x) = 0$, and use the graph to factor $V(x)$.

$$V(x) = -28(x-2)(x+3)$$



$$(x-2)$$

10. **Sports** – The manager of a basketball team charted the team's progress for the season. For each game, she took the team's points and subtracted the points that the other team scored. The team's performance can be modeled by the function $P(x) = x^3 - 9x^2 + 18x$, where x represents the number of games since the start of the season.

- a. Find the zeros of the function. What do they represent?

$$(x-0) \quad x(x^2 - 9x + 18)$$

$$x = 0, 3, 6$$

$$x = 0, 3, 6$$

- b. Write the function in factored form.

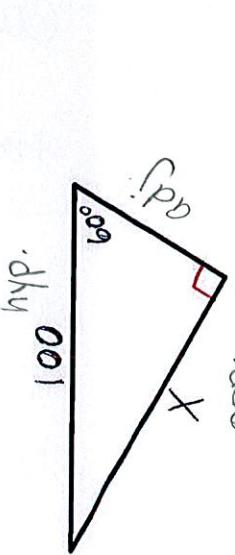
$$P(x) = (x-0)(x-3)(x-6)$$

$$P(x) = x(x-3)(x-6)$$

Trigonometric Ratios of Special Right Triangles

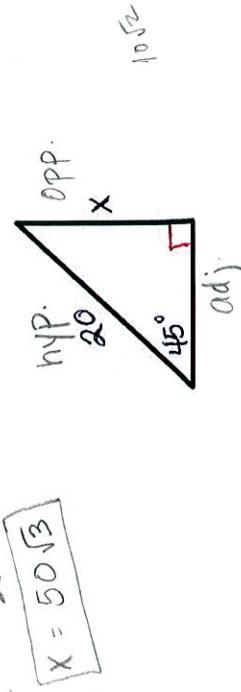
Diagram	Sine	Cosine	Tangent
	$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
	$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$
	$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\tan 45^\circ = \frac{1}{1} = 1$

Example 1: Finding Side Lengths of Special Right Triangles



Use a trigonometric function to find the value of x.

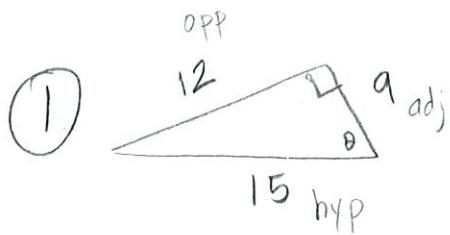
$$\text{Given } \sin 60^\circ = \frac{x}{100} \Rightarrow \frac{x}{100} = \frac{\sqrt{3}}{2} \Rightarrow x = \frac{100\sqrt{3}}{2} \Rightarrow x = 50\sqrt{3}$$



Example 2: Finding Side Lengths of Special Right Triangles

Use a trigonometric function to find the value of x.

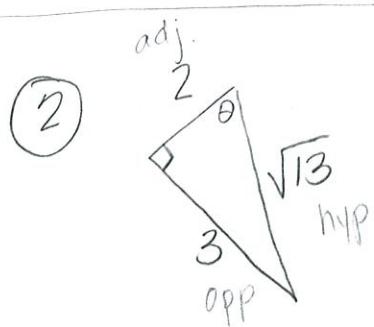
$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \sin 45^\circ = \frac{x}{20} \Rightarrow \frac{\sqrt{2}}{2} = \frac{x}{20} \Rightarrow \frac{2x}{2} = 20\sqrt{2} \Rightarrow x = 20\sqrt{2}$$



$$\sin \theta = \frac{12}{15} = \boxed{\frac{4}{5}}$$

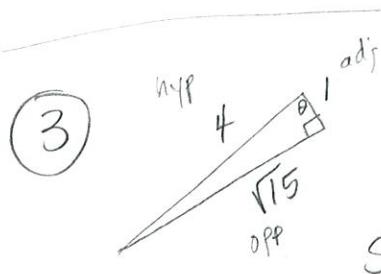
$$\cos \theta = \frac{9}{15} = \boxed{\frac{3}{5}}$$

$$\tan \theta = \frac{12}{9} = \boxed{\frac{4}{3}}$$



$$\sin \theta = \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{3\sqrt{13}}{13}}$$

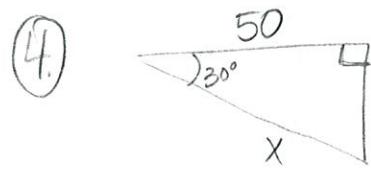
$$\cos \theta = \frac{2}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{2\sqrt{13}}{13}}$$



$$\sin \theta = \frac{\sqrt{15}}{4}$$

$$\cos \theta = \boxed{\frac{1}{4}}$$

$$\tan \theta = \frac{\sqrt{15}}{1} = \boxed{\sqrt{15}}$$



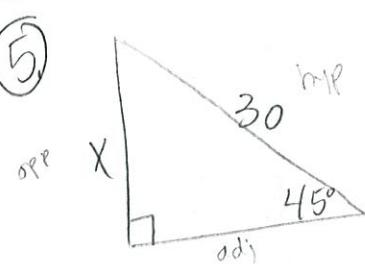
$$\cos 30^\circ = \frac{50}{x}$$

$$\frac{\sqrt{3}}{2} = \frac{50}{x}$$

$$\sqrt{3}x = \frac{100}{\sqrt{3}}$$

$$x = \frac{100}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{100\sqrt{3}}{3}}$$

≈ 57.74



$$\sin 45^\circ = \frac{x}{30}$$

$$\frac{\sqrt{2}}{2} = \frac{x}{30} \Rightarrow 2x = \frac{30\sqrt{2}}{2} \Rightarrow \boxed{x = 15\sqrt{2}}$$

≈ 21.21



$$\tan 60^\circ = \frac{250}{x}$$

$$\sqrt{3} = \frac{250}{x}$$

$$\frac{250}{\sqrt{3}} = \frac{\sqrt{3}x}{\sqrt{3}} \Rightarrow$$

$$\frac{250}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{250\sqrt{3}}{3}}$$

≈ 144.34

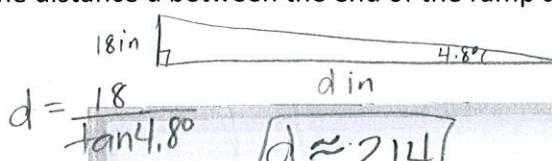
Trigonometric Lessons

Name: Cajuday notes

Example 3: Construction Application

A builder is constructing a wheelchair ramp from the ground to a deck with a height of 18 in.

The angle between the ground and the ramp must be 4.8° . To the nearest inch, what should be the distance d between the end of the ramp and the deck?



$$\tan \theta = \frac{\text{opp}}{\text{adj.}}$$

$$d (\tan 4.8^\circ) = 18$$

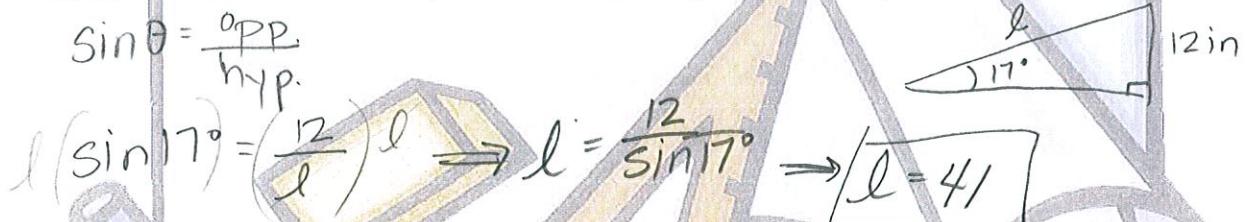
$$d (\tan 4.8^\circ) = \left(\frac{18}{d}\right) d$$

$$\sin 17^\circ = \frac{12}{l}$$

$$l = \frac{12}{\sin 17^\circ}$$

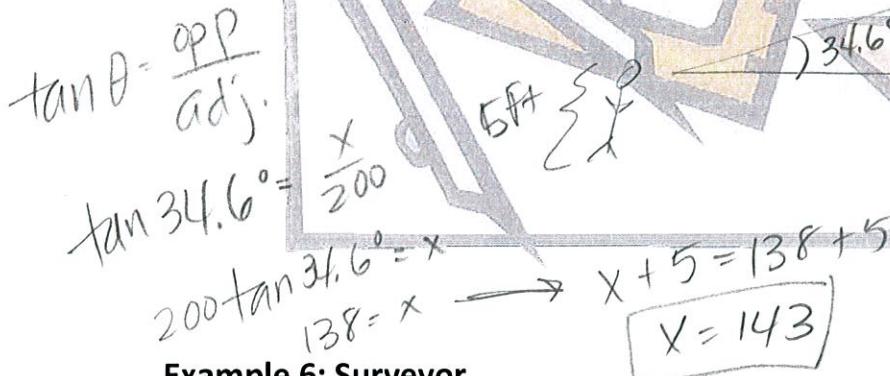
Example 4: Skateboard

A skateboard ramp will have a height of 12 in., and the angle between the ramp and the ground will be 17° . To the nearest inch, what will be the length l of the ramp?



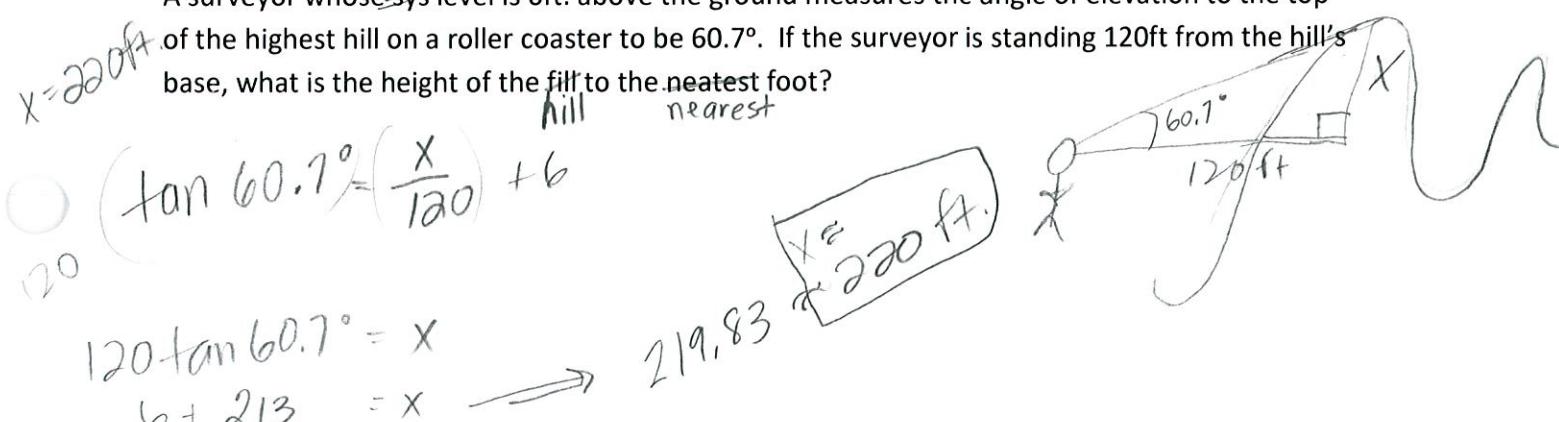
Example 5: Geology Application

A park ranger whose eye level is 5ft. above the ground measures the angle of elevation to the top of an eruption of Old Faithful geyser to be 34.6° . If the ranger is standing 200ft. from the geyser's base, what is the height of the eruption to the nearest foot?

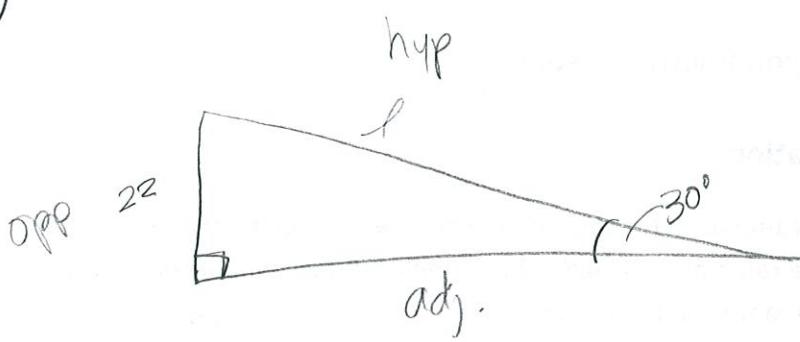


Example 6: Surveyor

A surveyor whose ~~eye~~ level is 6ft. above the ground measures the angle of elevation to the top of the highest hill on a roller coaster to be 60.7° . If the surveyor is standing 120ft from the hill's base, what is the height of the hill to the nearest foot?



①



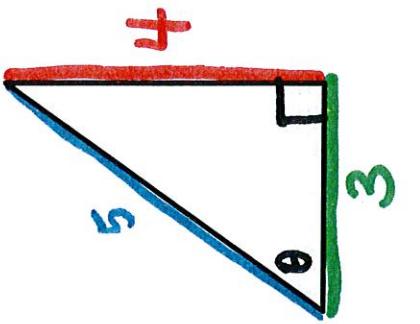
$$\sin 30^\circ = \left(\frac{22}{l}\right)l$$

$$\cancel{l} \left(\sin 30^\circ \right) = \frac{22}{\sin 30^\circ}$$

$$l = 44 \text{ ft}$$

Reciprocal Trigonometric Functions

Words	Numbers	Symbols
The cosecant (csc) of angle θ is the reciprocal of the sine function.	$csc \theta = \frac{5}{4}$	$csc \theta = \frac{hyp.}{opp.} = \frac{hyp.}{opp.}$
The secant (sec) of angle θ is the reciprocal of the cosine function.	$sec \theta = \frac{5}{3}$	$sec \theta = \frac{hyp.}{adj.} = \frac{hyp.}{adj.}$
The cotangent (cot) of angle θ is the reciprocal of the tangent function.	$cot \theta = \frac{3}{4}$	$cot \theta = \frac{adj.}{opp.} = \frac{adj.}{opp.}$



Example 1: Finding all Trigonometric Ratios

Find the values of the six trigonometric functions for θ .

$$\begin{aligned} a^2 + b^2 &= c^2 \\ C^2 &= 14^2 + 4^2 \\ C &= 2500 \\ C &= 50 \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{4}{50} = \frac{24}{25} \\ \cos \theta &= \frac{14}{50} = \frac{7}{25} \\ \tan \theta &= \frac{4}{7} = \frac{24}{24} \\ \csc \theta &= \frac{25}{24} \\ \sec \theta &= \frac{25}{7} \\ \cot \theta &= \frac{7}{24} \end{aligned}$$

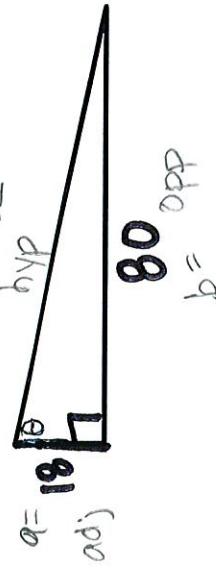
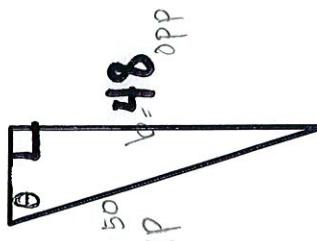
Example 2: Finding all Trigonometric Ratios

Find the values of the six trigonometric functions for θ .

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 18^2 + 80^2 &= c^2 \\ 324 + 6400 &= c^2 \\ 6724 &= c^2 \\ c &= 82 \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{40}{82} \\ \cos \theta &= \frac{9}{82} \\ \tan \theta &= \frac{40}{9} \\ \csc \theta &= \frac{82}{40} \\ \sec \theta &= \frac{82}{9} \\ \cot \theta &= \frac{9}{40} \end{aligned}$$

$$\begin{aligned} a &= 14 \text{ adj.} \\ a &= 14 \end{aligned}$$



Practice

Soh cah toa
cho sha cao

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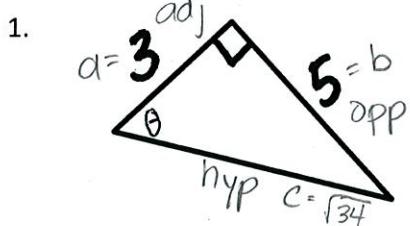
Cajudoy

Block: ALL

Date: 8/20/12 or 8/21/12

Find the values of the six trigonometric functions for θ .

$$\begin{aligned} \csc \theta &= \frac{\sqrt{34}}{5} \\ \sec \theta &= \frac{\sqrt{34}}{3} \\ \cot \theta &= \frac{3}{5} \end{aligned}$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ 3^2 + 5^2 &= c^2 \\ 9 + 25 &= c^2 \\ 34 &= c^2 \\ \sqrt{34} &= c \end{aligned}$$

$$\sin \theta = \frac{5}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{5\sqrt{34}}{34}$$

$$\cos \theta = \frac{3}{\sqrt{34}} \cdot \frac{\sqrt{34}}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$$

$$\tan \theta = \frac{5}{3}$$

2.

$$\cot \theta = \frac{2}{b} = \frac{1}{3} \quad a = 2$$

$$\begin{aligned} b &= 6 \text{ OPP} & \csc \theta &= \frac{2\sqrt{10}}{6} \\ a &= \sqrt{21} \text{ adj} & &= \frac{\sqrt{10}}{3} \end{aligned}$$

$$\begin{aligned} \csc \theta &= 2.5 \Rightarrow \frac{5}{2} \\ \sec \theta &= \frac{2.5}{\sqrt{5.25}} \text{ OPP} \\ &= \frac{2.5\sqrt{5.25}}{5.25} \\ \cot \theta &= \sqrt{5.25} \Rightarrow \sqrt{\frac{21}{4}} \end{aligned}$$

$$\begin{aligned} 2^2 + b^2 &= (2\sqrt{10})^2 \\ 4 + b^2 &= 40 \\ -4 & \quad -4 \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{3\sqrt{10}}{12\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} \\ &= \frac{3\sqrt{10}}{10} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{2}{12\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} \\ &= \frac{\sqrt{10}}{10} \end{aligned}$$

3.

$$b^2 + b^2 = 10^2 \quad 4.$$

$$36 + b^2 = 100$$

$$b^2 = 64$$

$$b = 8$$

$$\begin{aligned} a^2 &= 5.25 \frac{25}{4} \\ a &= \sqrt{\frac{21}{4}} \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{8}{10} = \frac{4}{5} \\ \cos \theta &= \frac{6}{10} = \frac{3}{5} \\ \tan \theta &= \frac{8}{6} = \frac{4}{3} \end{aligned}$$

$$\begin{aligned} \sec \theta &= \frac{2\sqrt{10}}{2} \\ &= \sqrt{10} \\ \sin &= \frac{1}{\sqrt{5.25}} \\ \cos &= \frac{\sqrt{21}}{\sqrt{5.25}} \\ &= \frac{\sqrt{5.25}}{2.5} \end{aligned}$$

$$\begin{aligned} \tan &= \frac{6}{2} = 3 \\ &= \frac{6}{2} = 3 \end{aligned}$$

$$\begin{aligned} \csc \theta &= \frac{1}{\frac{1}{\sqrt{5.25}}} \\ &= \sqrt{5.25} \\ &= \frac{\sqrt{5.25}}{5.25} \end{aligned}$$

5.

$$4^2 + 1^2 = C^2$$

$$16 + 1 = C^2$$

$$17 = C^2$$

$$\sqrt{17} = C$$

$$\begin{aligned} a &= 4 \text{ adj} \\ & \quad \quad \quad \text{opp} = b \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{1}{\sqrt{17}} = \frac{\sqrt{17}}{17} \\ \cos \theta &= \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17} \\ & \quad \quad \quad \csc \theta = \frac{\sqrt{17}}{4} \\ \tan \theta &= \frac{1}{4} \end{aligned}$$

6.

$$a^2 + 4^2 = (4\sqrt{2})^2$$

$$a^2 + 16 = 32$$

$$a^2 = 16 \quad a = 4$$

$$\begin{aligned} \sin \theta &= \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \cos \theta &= \frac{4}{4\sqrt{2}} = \frac{\sqrt{2}}{2} \\ \tan \theta &= \frac{4}{4} = 1 \end{aligned}$$

$$\begin{aligned} \csc \theta &= \frac{\sqrt{2}}{2} \\ \sec \theta &= \sqrt{2} \\ \cot \theta &= \frac{4}{4} = 1 \end{aligned}$$

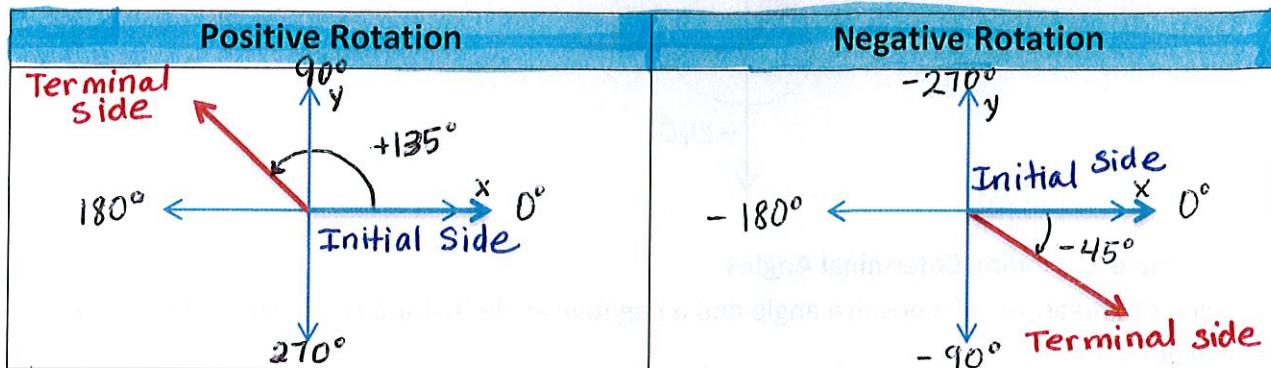
Angles of Rotation



Why learn this?

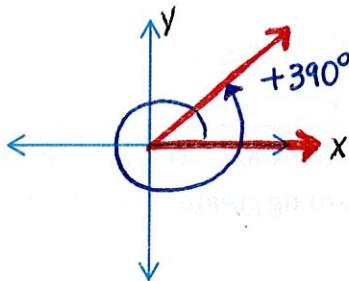
You can use angles of rotation to determine the rate at which a skater must spin to complete a jump.

An angle is in Standard position when its vertex is at the origin and one ray is on the positive x-axis. The initial side of the angle is the ray on the x-axis. The other ray is called the terminal side of the angle.



An angle of rotation is formed by rotating the terminal side and keeping the initial side in place. If the terminal side is rotated counterclockwise, the angle of rotation is positive. If the terminal side is rotated clockwise, the angle of rotation is negative. The terminal side can be rotated more than 360° .

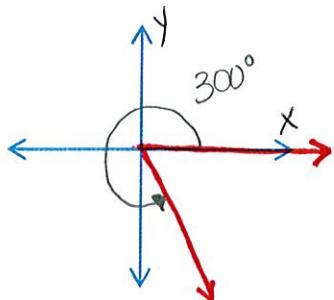
Example: $+390^\circ$



Example 1: Drawing Angles in Standard Position

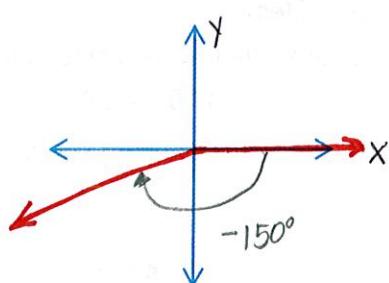
Draw an angle with the given measure in standard position.

A. 300°



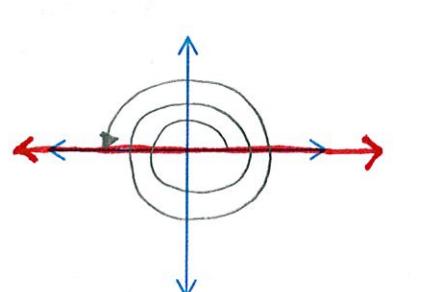
counter clockwise

B. -150°



Clockwise

C. 900°



counter clockwise

$$900^\circ - 360^\circ = 540^\circ$$

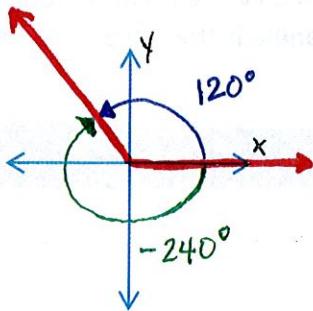
$$900^\circ - 360^\circ = 180^\circ$$

$$540^\circ - 360^\circ = 180^\circ$$

$$360^\circ + 360^\circ + 180^\circ$$

Coterminal angles are angles in standard position with the same terminal side. For example, angles measuring 120° and -240° are coterminal.

There are infinitely many coterminal angles. One way to find the measure of an angle that is coterminal with an angle θ is to add or subtract integer multiples of 360° .



Video

Example 2: Finding Coterminal Angles

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

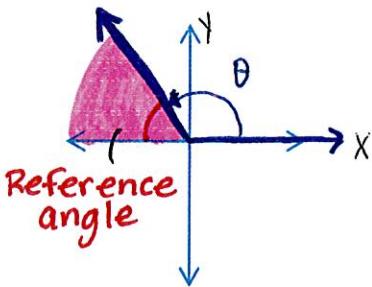
A. $\theta = 40^\circ$ $40^\circ + 360^\circ = 400^\circ$ positive coterminal angle.

~~$40^\circ - 360^\circ = -320^\circ$~~ negative coterminal angle.

B. $\theta = 380^\circ$ $380^\circ - 360^\circ = 20^\circ$ ~~40°~~ positive coterminal angle

$380^\circ - 2(360^\circ) = -340^\circ$ OR $-360^\circ + 20^\circ = -340^\circ$ NCA

For an angle θ in standard position, the reference angle is the positive acute angle formed by the terminal side of θ and the x-axis. You will learn how to use reference angles to find trigonometric values of angles measuring greater than 90° or less than 0° .



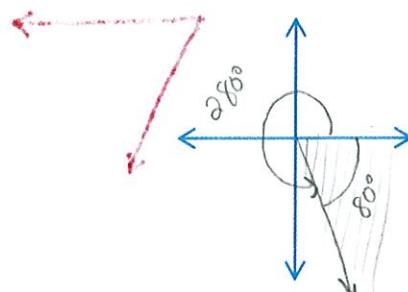
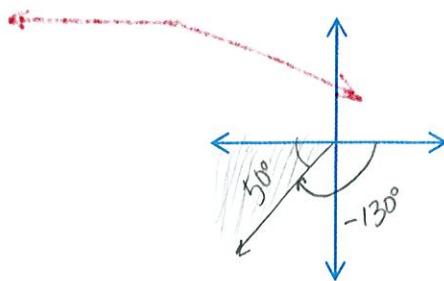
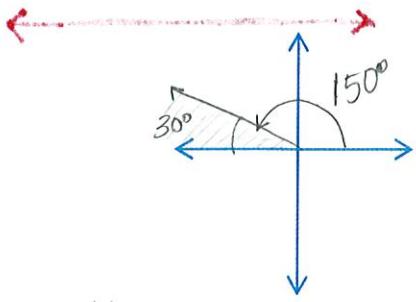
Example 3: Finding Reference Angles

Find the measure of the reference angle for each given angle.

A. $\theta = 150^\circ$

B. $\theta = -130^\circ$

C. $\theta = 280^\circ$



B2E stopped here

The distance r from the point P to the origin

Trigonometric Functions

For a point $P(x, y)$ on the terminal side of θ in standard position and $r = \sqrt{x^2 + y^2}$

Sine	Cosine	Tangent
$\sin \theta = \frac{y}{r}$	$\cos \theta = \frac{x}{r}$	$\tan \theta = \frac{y}{x}$ $x \neq 0$

Example 4: Finding Values of Trigonometric Functions

$P(4, -5)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ .

① Plot point P , and use it to sketch a right triangle and angle. find r .

$$r = \sqrt{4^2 + (-5)^2}$$

$$r = \sqrt{16 + 25}$$

$$r = \sqrt{41}$$

② find $\sin \theta, \cos \theta$, and $\tan \theta$

$$\sin \theta = \frac{-5}{\sqrt{41}} = -\frac{5\sqrt{41}}{41}$$

$$\cos \theta = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$$

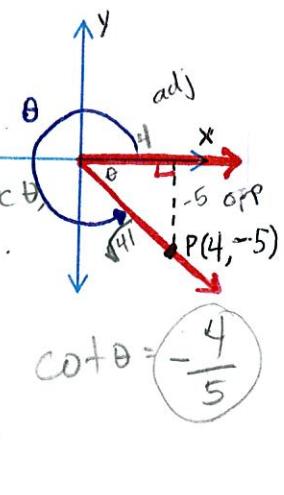
$$\tan \theta = -\frac{5}{4} = -\frac{5}{4}$$

③ reciprocals
 $\csc \theta, \sec \theta$
 $\text{and } \cot \theta$.

$$\csc \theta = \frac{\sqrt{41}}{5}$$

$$\sec \theta = \frac{\sqrt{41}}{4}$$

$$\cot \theta = -\frac{4}{5}$$



GET ORGANIZED: In each box, describe how to determine the given angle or position for an angle θ .

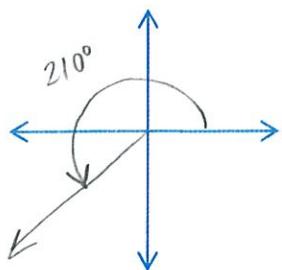
Standard position	Angle θ 250°	Reference angle
$250^\circ + 360^\circ = 610^\circ$	$250^\circ - 360^\circ = -110^\circ$	
Positive coterminal angle	Negative coterminal angle	

DRAW 2 go left & right
 have neighbor give you an angle.

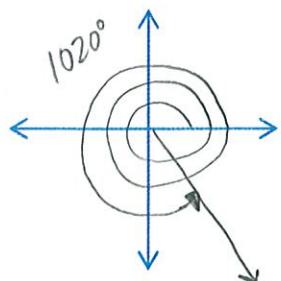
Practice:

Draw an angle with the given measure in standard position.

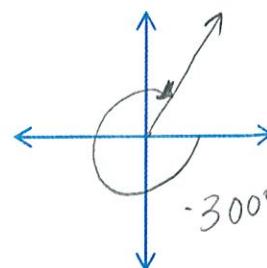
1. 210°



2. 1020°



3. -300°



$$\begin{array}{r}
 360 \\
 +180 \\
 \hline
 540
 \end{array}
 \quad
 \begin{array}{r}
 360 \\
 -120 \\
 \hline
 240
 \end{array}$$

$$\begin{array}{r}
 180 \\
 -40 \\
 \hline
 +140
 \end{array}$$

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

4. $\theta = 88^\circ$

$$88^\circ + 360^\circ = 448^\circ \text{ PCA}$$

$$88^\circ - 360^\circ = -272^\circ \text{ NCA}$$

5. $\theta = 500^\circ$

$$500^\circ + 360^\circ = 860^\circ$$

$$500^\circ - 360^\circ = 140^\circ$$

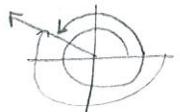
$$500^\circ - 2(360^\circ) = -220^\circ$$

$$-360^\circ + 140^\circ = -220^\circ$$

6. $\theta = -120^\circ$

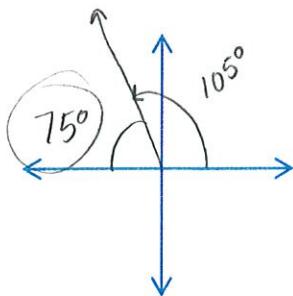
$$-120^\circ + 360^\circ = 240^\circ$$

$$-120^\circ - 360^\circ = -480^\circ$$

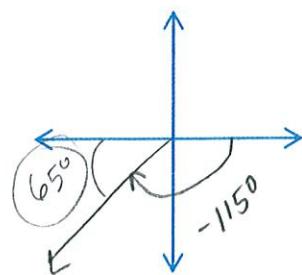


Find the measure of the reference angle for each given angle.

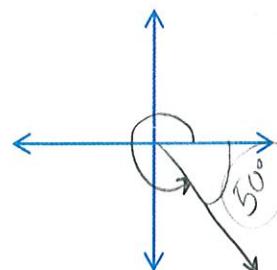
7. $\theta = 105^\circ$



8. $\theta = -115^\circ$



9. $\theta = 310^\circ$



$$\begin{array}{r}
 180 \\
 -115 \\
 \hline
 65
 \end{array}$$

~~$$\begin{array}{r}
 360 \\
 -310 \\
 \hline
 50
 \end{array}$$~~

10. $P(-3, 6)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ .

$$\sin \theta = \frac{3\sqrt{5}}{5}$$

$$\csc \theta = \frac{\sqrt{5}}{2}$$

$$r = \sqrt{45}$$

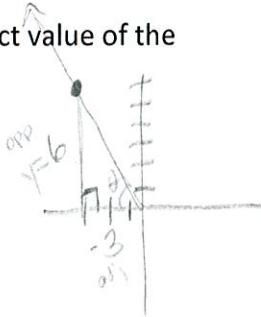
$$\cos \theta = \frac{\sqrt{5}}{5}$$

$$\sec \theta = -\sqrt{5}$$

$$r = 3\sqrt{5}$$

$$\tan \theta = -2$$

$$\cot \theta = -\frac{1}{2}$$



Unit Circle Practice:

Convert each measure from degrees to radians or from radians to degrees.

1. 80°

$$80^\circ \cdot \left(\frac{\pi}{180^\circ}\right) = \frac{8\pi}{18} = \frac{4\pi}{9}$$

2. $\frac{2\pi}{9}$ radians

$$\frac{2\pi}{9} \left(\frac{180^\circ}{\pi}\right) = 2 \cdot 20 = 40^\circ$$

3. -36°

$$-36^\circ \left(\frac{\pi}{180^\circ}\right) = -\frac{36\pi}{180} = \frac{6\pi}{30} = \frac{\pi}{5}$$

4. 4π radians

$$4\pi \left(\frac{180^\circ}{\pi}\right) = 720^\circ$$

Use the unit circle to find the exact value of each trigonometric function.

5. $\sin 315^\circ$

$$-\frac{\sqrt{2}}{2}$$

6. $\tan 180^\circ$

$$\frac{0}{-1} = 0$$

7. $\cos \frac{4\pi}{3}$

$$-\frac{1}{2}$$

$$\tan 6^\circ = .103104$$

Use reference angle to find the exact value of the sine, cosine, and tangent of each angle.

8. 270°

Reference angle 90°

$$\sin 270^\circ = -1$$

$$\cos 270^\circ = 0$$

$$\tan 270^\circ = \frac{1}{0} \text{ undefined}$$

9. $\frac{11\pi}{6}$

$$RA = 30^\circ$$

$$\sin \frac{11\pi}{6} = \frac{1}{2}$$

$$\cos \frac{11\pi}{6} = \frac{\sqrt{3}}{2}$$

10. -30°

$$\sin -30^\circ = -\frac{1}{2}$$

$$\cos -30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan -30^\circ = -\frac{\sqrt{3}}{3}$$

11. An hour hand on Big Ben's Clock Tower in London is 14 ft long. To the nearest tenth of a foot, how far does the tip of the hour hand travel in 1 minute?

① radius = $14 \text{ ft} \cdot (2) = 28 \text{ ft}$

③ $S = r\theta$

$$S = 28 \left(\frac{\pi}{360}\right)$$

$$S = .24434 \dots$$

$$\frac{1}{6} \cdot 120\pi = \frac{720 \cdot \theta}{720}$$

$$\frac{2\pi}{720} = \theta = \frac{\pi}{360}$$

$$1 \text{ min} = \frac{1}{60} \times 6^\circ$$

$$\tan 6^\circ = \frac{14}{107.14}$$

Kruas



Who uses this?

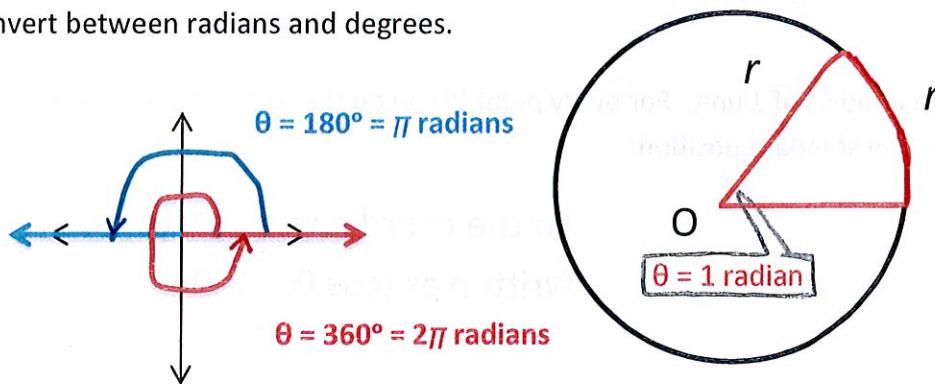
Engineers can use angles measured in radians when designing machinery used to train astronauts.



So far, you have measured angles in degrees. You can also measure angles in radians.

A **radian** is a unit of angle measure based on arc length. Recall from geometry that an *arc* is an unbroken part of a circle. If a central angle θ in a circle of radius r intercepts an arc of length r , then the measure of θ is defined as 1 radian.

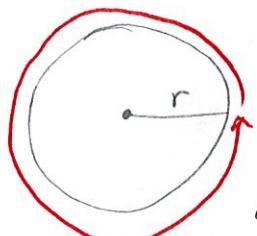
The circumference of a circle of radius r is $2\pi r$. Therefore, an angle representing one complete clockwise rotation measures 2π radians. You can use the fact that 2π radians is equivalent to 360° to convert between radians and degrees.



Converting Angle Measures

Degrees to Radians	Radians to Degrees
Multiply number of degrees by $x^\circ \left(\frac{\pi \text{ radians}}{180^\circ} \right)$	Multiply the number of radians by $x \text{ radians} \left(\frac{180^\circ}{\pi \text{ radians}} \right)$

Video Notes:



$$\begin{aligned} \text{Circumference} &= \\ 2\pi r &= \text{radians!} = 360^\circ \\ \pi \text{ radians} &= 180^\circ \end{aligned}$$

$$-30^\circ \left(\frac{\pi \text{ radians}}{180^\circ} \right)$$

$$-\frac{3\pi}{18} = \boxed{-\frac{\pi}{6} \text{ radians}}$$

$$\frac{5\pi}{9} \left(\frac{180^\circ}{\pi \text{ radians}} \right)$$

$$= \frac{5 \cdot 20}{\pi} = \boxed{100^\circ}$$

Example 1: Converting Between Degrees and Radians

Converting each measure from degrees to radians or from radians to degrees.

$$A. -45^\circ \left(\frac{\pi \text{ radians}}{180^\circ} \right) = \frac{-45}{1} \cdot \frac{\pi}{180^\circ} \Rightarrow \frac{-45\pi}{180^\circ} = \boxed{\frac{-\pi}{4} \text{ radians}}$$

$$B. \frac{5\pi}{6} \text{ Radians} \left(\frac{180^\circ}{\pi \text{ radians}} \right) = \frac{900}{6} = \boxed{150^\circ}$$

The unit circle is a circle with a radius of 1 unit. For every point P(x, y) on the unit circle, the value of r is 1. Therefore, for an angle θ in standard position:

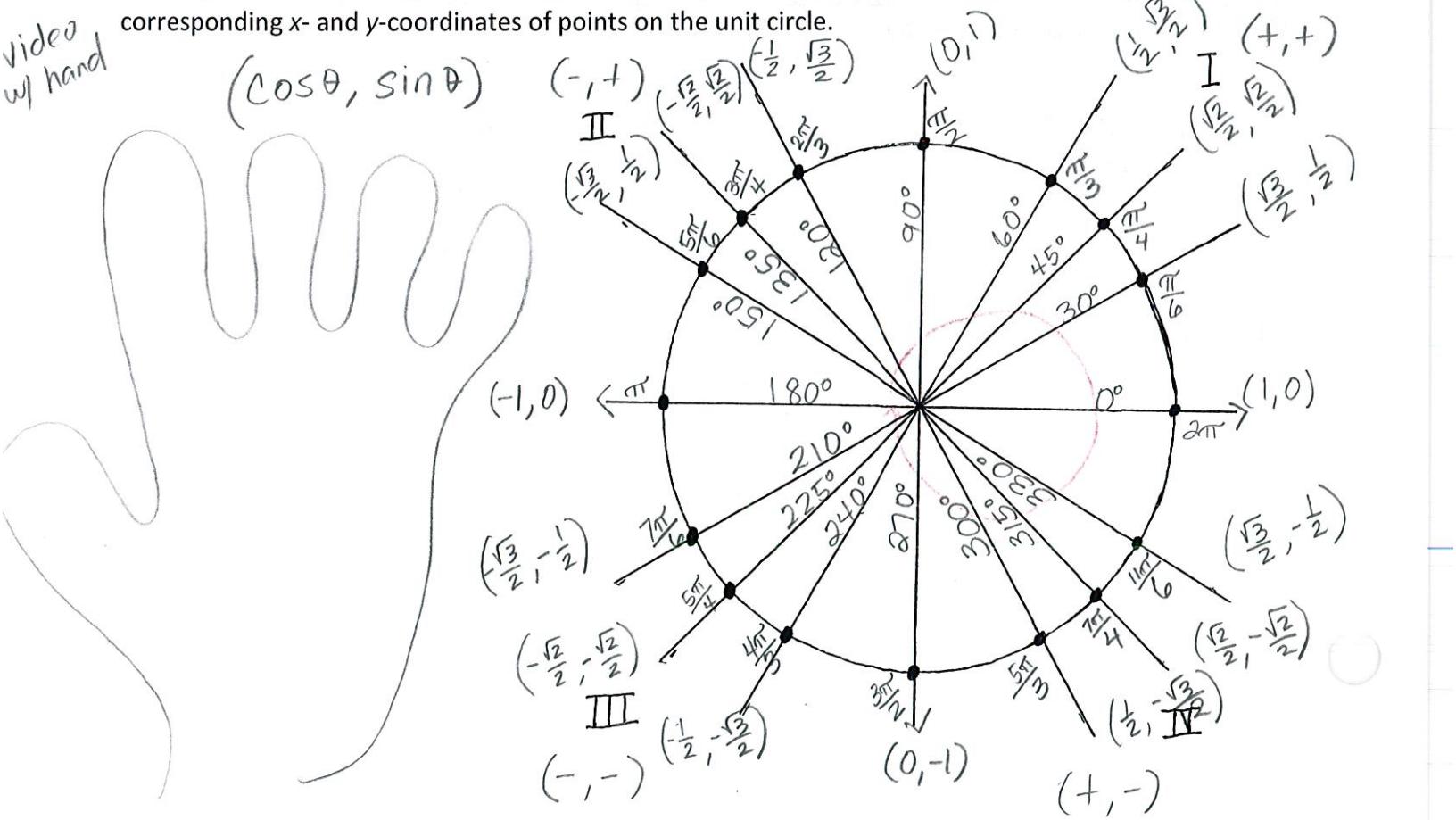
$$\sin \theta = \frac{y}{r} = \frac{y}{1} = y$$

$$\cos \theta = \frac{x}{r} = \frac{x}{1} = x$$

$$\tan \theta = \frac{y}{x}$$

So the coordinates of P can be written as $(\cos \theta, \sin \theta)$.

The diagram shows the equivalent degree and radian measures of special angles, as well as the corresponding x- and y-coordinates of points on the unit circle.



Example 2: Using the Unit Circle to find the exact value of each trigonometric function.

A. $\cos 210^\circ = \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) \Rightarrow \cos 210^\circ = x$

passes through this $\rightarrow x = -\frac{\sqrt{3}}{2}$

B. $\tan \frac{5\pi}{3} \Rightarrow \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$\tan \frac{5\pi}{3} = \frac{y}{x} \Rightarrow \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \cdot \frac{2}{1} = -\sqrt{3}$

Trigonometric Functions and Reference Angles

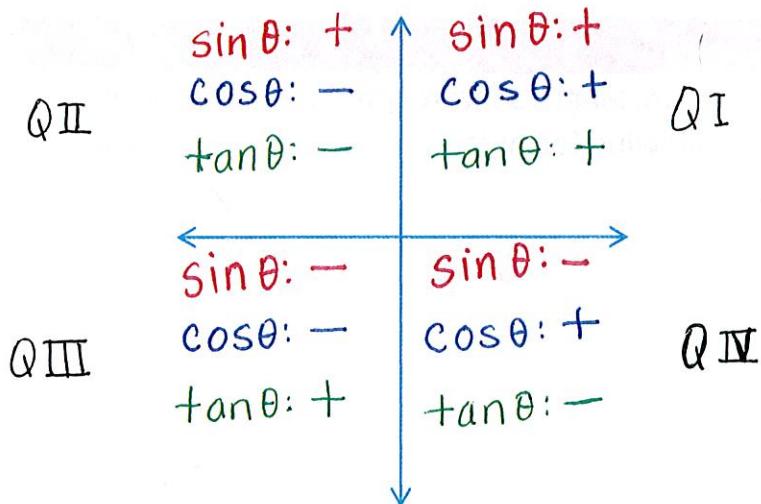
To find the sine, cosine, or tangent of θ :

Step 1: Determine the measure of the reference angle of θ .

Step 2: Use Quadrant I of the unit circle to find the sine, cosine, or tangent of the reference angle.

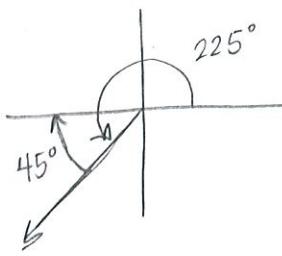
Step 3: Determine the quadrant of the terminal side of θ in standard position. Adjust the sign of the sine, cosine, or tangent based upon the quadrant of the terminal side.

The diagram shows how the signs of the trigonometric functions depend on the quadrant containing the terminal side of θ in standard position.



Example 3: Using Reference Angles to Evaluate Trigonometric Functions

Use a reference angle to find the exact value of the sine, cosine, and tangent of 225° .



① Find reference angle.

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = 1$$

$$\sin 225^\circ = -\frac{\sqrt{2}}{2}$$

$$\cos 225^\circ = -\frac{\sqrt{2}}{2}$$

$$\tan 225^\circ = 1$$

A. 120°

reference angle
 60°

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \sqrt{3} \Rightarrow \frac{\sqrt{3}}{1/2} = \frac{\sqrt{3}}{2} \cdot \frac{1}{1} = \sqrt{3}$$

B. $\frac{\pi}{3}$ reference angle is it self.
 $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$$\cos \frac{\pi}{3} = \frac{1}{2}$$

$$\tan \frac{\pi}{3} = \sqrt{3}$$

If you know the measure of a central angle of a circle, you can determine the lengths of the arc intercepted by the angle.

$$\frac{\text{radian measure of } \theta}{\text{radian measure of circle}} \rightarrow \frac{\theta}{2\pi} = \frac{s}{2\pi r} \leftarrow \frac{\text{arc length intercepted by } \theta}{\text{arc length intercepted by circle}}$$

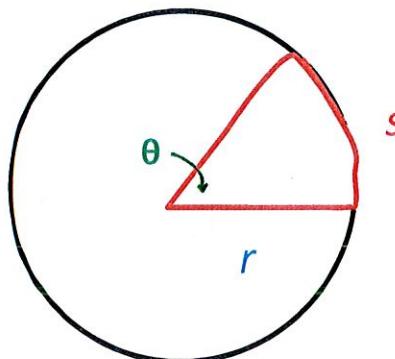
$$\theta = \frac{s}{r} \quad \text{Multiply each side by } 2\pi.$$

$$s = r\theta \quad \text{Solve for } s.$$

Arc Length Formula

For a circle of radius r , the arc length s intercepted by a central angle θ (measured in radians) is given by the following formula.

$$s = r\theta$$



Example 4: Engineering Application

A human centrifuge is a device used in training astronauts. The passenger cab of the centrifuge shown makes 32 complete revolutions about the central hub in 1 minute. To the nearest foot, how far does an astronaut in the cab travel in 1 second?

① Find radius

$$r = \frac{58}{2} = 29 \text{ ft.}$$

② Find the angle θ through which the cab rotates in 1 second

$$\theta \text{ radians} = \frac{32(2\pi) \text{ radians}}{60 \text{ s}}$$

Cross multiply

$$\frac{60 \cdot \theta}{60} = \frac{32(2\pi)}{60}$$

$$\theta = \frac{32(2\pi)}{60}$$

$$\theta = \frac{64\pi}{60}$$

$$\boxed{\theta = \frac{16\pi}{15}}$$

③ find the length of the arc

$$S = r\theta$$

$$S = 29 \left(\frac{16\pi}{15} \right)$$

$$S \approx 97$$

An astronaut travels about 97 feet in 1 second.



Passenger
cab

