

Complex Number Maze

Complete the maze by simplifying each expression. Simplify each expression and shade in the squares with imaginary numbers. You will have a path leading from the start square to the end square.

$(1+i)(1-i)$	$(2+3i) + (-4+5i)$	$(5-6i)(6-2i)$	$2i(3i^2)$	$3i(2i)$	Start Here $\sqrt{-4}$
$1 - i + i - i^2$ 1 + 1	$2 + 8i$	$30 - 10i - 36i$ $30 - 46i + 12i$ $30 - 46i - 12$	$4i$ $-6i$	$6i^2$ $i^2 = -1$ -6	$2i$
$\sqrt{5-4}$ 1	$-\sqrt{-49}$ $-7i$	$(3+2i) - (4+2i)$ -1	$\sqrt{-36}$ $6i$	$\sqrt{-25} + 3$ $5i + 3$	$2(3+2i)$ $6+4i$
$\sqrt{\frac{81}{25}}$ $\frac{9}{5}$	$(5+14i) - (10+2i)$ $-5 + 16i$	$(5+4i) - (+1+2i)$ $6+6i$	$3 + \sqrt{5}$	$-\sqrt{64}$ -8	$2i - (3+2i)$ -3
$(2+3i)(2-3i)$ $4 - 6i + 6i - 9i^2$ 13 4 + 9	$5i - \sqrt{-25}$ $5i - 5i$ 0	$(3+4i)(4-3i)$ $12 - 9i + 11i - 12i^2$ $24 + 7i$	$4 - \sqrt{-25}$ $4 - 5i$	$-\sqrt{-4}$ $-2i$	$3i(2+3i)$ $6i + 9i^2$ $6i - 9$
$(6+2i) + (1-2i)$ 7	i^2 = -1	$\sqrt{125}$ $5\sqrt{5}$	$4i^2$ -4	$(1-3i)(1+3i)$ $1+3i - 3i - 9i^2$ 1 + 9 10	$(1+2i)(-1-2i)$ $-1-2i - 2i - 4i^2$ $-1-4i + 4$ $i - 4i$
$\sqrt{-225}$ 15i	$(5+4i) - (1+2i)$ $4+2i$	$(1+2i) + (2-3i)$ -1i	$(2i^2)(-3i^2)$ -2 3 -16	$2(3+4i)$ $6+8i$	$(6+2i)(3i)$ $18i + 6i^2$ $18i - 6$
$-\sqrt{-1}$ -i	$-3i(-5i)$ $15i^2$ 15	$5i^2(2+i)$ -5 $10 - 5i$	$(2-3i) - 3i$ $9 - 6i$ $i + 6i$	$3i(2-i)$ $3 - 2i$ $1 + i$	$-\sqrt{625}$ -25
End Here					

$-6i + 6i$
 $-9i - 9i$

Name: _____

Cayley

 $\frac{38+8i}{10}$

Complex Expressions

Simplify the following expressions using properties of complex numbers.
Express your answer in standard form ($a + bi$).

1. $(4 + 6i) + (12 - 2i)$

$16 + 4i$

4. $(3 + 2i)^2$

$(3 + 2i)(3 + 2i) = 9 + 6i + 6i + 4i^2 = 5 + 12i$

7. $i(-1 + 15i)$

$-i - 15$

10. $(11 - i) - (-11 - i)$

22

13. $4(4 + 5i)$

$16 + 20i$

16. $(6i)(8i^2)(-3i)(4i^5)$

$27 - 9i^2 + 18i - 9i^3 = 36 + 9i$

19. $\frac{9 + 9i}{3 + i^2} = \frac{(3 - i^2)}{(3 - i^2)} = \frac{36 + 36i}{8}$

22. $(3 - 5i) + (11 + 16i)$

$14 + 11i$

25. $(-5 - 3i)(5 + 3i)$

$-25 - 15i - 15i - 9i^2 = 9 - 30i$

28. $\left(\frac{2}{5} + \frac{4}{5}i\right)^2$

$-48 + 16i$

$2 + 8i - 3i - 12i = 14 + 5i$

$9. 8i + 10$

$10 + 8i$

2. $2i(3 + 12i)$

$6i + 24i^2 = 6i - 24 - 24 + 6i = -24 + 12i$

5. $(7 + 9i) - (5 - 2i)$

$2 + 11i$

8. $(21 + i)(3 - 3i)$

$43 - 63i + 3i - 3i^2 = 43 - 60i$

11. $(2 + 8i)^3 = -316 - 416i$

$(2 + 8i)(2 + 8i)(2 + 8i)$

14. $(2 - i)^2$

$(2 - i)(2 - i) = 4 - 4i + i^2 = 4 - 4i - 1 = 3 - 4i$

17. $(5 + i^3)^2 = (5 - i)^2 = 25 - 10i + i^2 = 24 - 10i$

20. $\frac{-2 + 3i}{-1 + 4i} = \frac{(-1 - 4i)}{(-1 - 4i)} = 1 + 4i - 4i - 16i^2 = 17 - 8i$

23. $(6 + 10i)^3 = 216 + 1080i - 1080i - 1000i^2 = 36 + 1000$

26. $i^2(-8 + 2i) = -8i^2 + 2i^3 = 8 - 2i$

29. $6(6 + 6i) = 36 + 36i$

$\frac{1}{6 + 10i} = \frac{(6 - 10i)}{(6 - 10i)} = \frac{36 - 60i + 60i - 100i^2}{36 + 100} = \frac{136}{136} = 1$

30. $2i(7 + i) + (6 + 3i) = 14i + 2i^2 + 6 + 3i = 14i - 2 + 6 + 3i = 4 + 17i$

$i^{31} = -i$

$2 - 34 = \frac{-34 + 18i}{10} = \frac{-34}{10} + \frac{18i}{10}$

3. $i^{20} \times i^{11}$

$2 + 6i + 12i + 36i^2 = -1 + \frac{-17}{5} + \frac{9i}{5}$

6. $\frac{2 + 12i}{1 - 3i} = \frac{(1 + 3i)}{(1 + 3i)} = \frac{-34 + 18i}{10}$

1. $1 + 3i - 3i - 9i^2 = 10$

9. $(4i + 6) + (4i + 4)$

$42 + 12i - 7i - 2i^2 = 42 + 10i$

12. $\frac{6 - i}{7 - 2i} = \frac{(7 + 2i)}{(7 + 2i)} = \frac{44 + 5i}{53} = \frac{44}{53} + \frac{5i}{53}$

$49 + 14i - 14i - 4i^2 = 49 + 4 = 53$

15. $(-9 + 8i) + (-14 + 8i)$

$-23 + 16i$

18. $\left(\frac{3}{4} + \frac{1}{4}i\right) - \left(\frac{3}{8} + \frac{5}{16}i\right)$

on board

21. $(-2 + 3i) - (2 + 3i)$

-4

24. $(-3 + 2i)^2(i^3) = -12 - 5i$

$7 - 10i$

27. $(5 - 12i) + (2 + 2i)$

$14i + 2i^2 = 14i - 2 + 6 + 3i$

30. $2i(7 + i) + (6 + 3i)$

$\frac{6 - 10i}{136} = \frac{6}{136} - \frac{10i}{136}$

$3 - \frac{5i}{18}$

$0.44 - 0.074i$

Name: Cajudoy

Solving Quadratic Equations

Use the quadratic formula to solve the following equations. Show your work and simplify your answers

$$1. \frac{3x^2 + 3x + 8}{a b c} = 0 \quad \frac{-3 \pm \sqrt{3^2 - 4(3)(8)}}{2(3)} \Rightarrow \frac{-3 \pm \sqrt{-87}}{6} \Rightarrow \boxed{\frac{-1 \pm i\sqrt{87}}{2}}$$

$$2. \frac{-2x^2 + 4x - 1}{a b c} = 0 \quad \frac{-4 \pm \sqrt{4^2 - 4(-2)(-1)}}{2(-2)} \Rightarrow \frac{-4 \pm \sqrt{8}}{-4} \Rightarrow \boxed{\frac{+1 \pm 2\sqrt{2}}{2}}$$

$$3. \frac{9x^2 + 5x + 2}{a b c} = 0 \quad \frac{-5 \pm \sqrt{5^2 - 4(9)(2)}}{2(9)} \Rightarrow \boxed{\frac{-5 \pm \sqrt{-47}}{18}} = \boxed{\frac{-5 \pm i\sqrt{47}}{18}}$$

$$4. \frac{6x^2 + 6x + 3}{a b c} = 0 \quad \frac{-6 \pm \sqrt{6^2 - 4(6)(3)}}{2(6)} \Rightarrow \frac{-6 \pm \sqrt{-36}}{12} \Rightarrow \frac{-6 \pm 6i}{12} = \boxed{\frac{-1 \pm i}{2}}$$

$$5. \frac{x^2 + 7x + 13}{a b c} = 0 \quad \frac{-7 \pm \sqrt{7^2 - 4(1)(13)}}{2(1)} \Rightarrow \boxed{\frac{-7 \pm i\sqrt{3}}{2}}$$

$$6. \frac{2x^2 + 8x + 6}{a b c} = 0 \quad \frac{-8 \pm \sqrt{8^2 - 4(2)(6)}}{2(2)} \Rightarrow \frac{-8 \pm \sqrt{16}}{4} \Rightarrow \frac{-8 \pm 4}{4} \Rightarrow \begin{aligned} x &= -1 \text{ OR} \\ x &= -3 \end{aligned}$$

$$7. \frac{-6x^2 + 9x - 4}{a b c} = 0 \quad \frac{-9 \pm \sqrt{9^2 - 4(-6)(-4)}}{2(-6)} \Rightarrow \frac{-9 \pm \sqrt{-15}}{-12} = \boxed{\frac{-3 \pm i\sqrt{15}}{-4}} \quad \boxed{\frac{3}{4} \pm \frac{\sqrt{15}}{4}i}$$

$$8. \frac{2x^2 + 10x + 13}{a b c} = 0 \quad \frac{-10 \pm \sqrt{10^2 - 4(2)(13)}}{2(2)} \Rightarrow \frac{-10 \pm \sqrt{-4}}{4} \Rightarrow \frac{-10 \pm 2i}{4} = \boxed{\frac{-5 \pm i}{2}}$$

$$9. \frac{-6x^2 + 11x - 15}{a b c} = 0 \quad \frac{-11 \pm \sqrt{11^2 - 4(-6)(-15)}}{2(-6)} \Rightarrow \frac{-11 \pm \sqrt{-239}}{-12} \Rightarrow \boxed{\frac{-11 \pm i\sqrt{239}}{-12}}$$

$$10. \frac{10x^2 + 13x + 12}{a b c} = 0 \quad \frac{-13 \pm \sqrt{13^2 - 4(10)(12)}}{2(10)} \Rightarrow \frac{-13 \pm \sqrt{-311}}{20} \Rightarrow \boxed{\frac{-13 \pm i\sqrt{311}}{20}}$$

Practice:

Express each number in terms of i .

1. $3\sqrt{-16}$ 12i

$$\begin{aligned}3 \cdot \sqrt{1} \cdot \sqrt{16} \\3 \cdot i \cdot 4\end{aligned}$$

2. $-\sqrt{-75}$ $5i\sqrt{3}$

$$\begin{aligned}-\sqrt{1} \cdot \sqrt{75} \\-\sqrt{25 \cdot 3}\end{aligned}$$

3. $\sqrt{-12}$ $2i\sqrt{3}$

$$\begin{aligned}\sqrt{1} \cdot \sqrt{3 \cdot 4} \\i \cdot \sqrt{3} \cdot 2\end{aligned}$$

4. $2\sqrt{-36}$ 12i

$$\begin{aligned}2 \cdot \sqrt{1} \cdot \sqrt{36} \\2 \cdot i \cdot 6\end{aligned}$$

Solve each equation.

5. $x^2 = -81$ $x = 9i$

$$\sqrt{x^2} = \sqrt{-81}$$

$$x = \sqrt{-1} \cdot \sqrt{81}$$

$$x = i \cdot 9$$

6. $3x^2 + 75 = 0$ $x = 5i$

$$\begin{aligned}3x^2 = -75 \\3x^2 = \frac{-75}{3} \\x^2 = -25\end{aligned}$$

7. $9x^2 + 25 = 0$

$$\begin{aligned}9x^2 = -25 \\9x^2 = \frac{-25}{9}\end{aligned}$$

$$x^2 = -\frac{25}{9}$$

$$x = i\sqrt{2.77}$$

Add or subtract. Write the results in the form $a + bi$.

8. $(-2 + 4i) + (3 - 11i)$

1 - 7i

9. $(4 - i) - (5 + 8i)$

$$4 - i - 5 - 8i$$

-1 - 9i

10. $(6 - 2i) + (-6 + 2i)$

0

11. $(10 + 3i) - (10 - 4i)$

7i

Multiply. Write the result in the form $a + bi$.

12. $2i(3 - 5i)$

$$\begin{aligned}6i - 10i^2 \\-1 \\10 + 6i\end{aligned}$$

13. $(5 - 6i)(4 - 3i)$

$$\begin{aligned}20 - 15i - 24i + 18i^2 \\20 - 18\end{aligned}$$

2 - 39i

14. $(4 - 4i)(6 - i)$

$$\begin{aligned}24 - 4i - 24i + 4i^2 \\24 - 4\end{aligned}$$

4(5 - 7i)

20 - 28i OR

17. $\frac{3-i}{2-i} \frac{(2+i)}{(2+i)} = \frac{(6+3i)-2i-i^2}{4+2i-2i-i^2}$

Dividing Complex Numbers

15. $\frac{3+7i}{8i} \frac{(-8i)}{(-8i)}$

$$\begin{aligned}-24i - 56i^2 \\-64i^2 \\-3i + 7 \\8\end{aligned}$$

16. $\frac{5+i}{2-4i} \frac{(2+4i)}{(2+4i)}$

$$\begin{aligned}10 + 20i + 2i + 4i^2 \\4 + 8i - 8i - 16i^2 \\4 + 16 \\6 + 22i \\20 \\3 + 11i \\10\end{aligned}$$

$$17. \frac{3-i}{2-i} \frac{(2+i)}{(2+i)} = \frac{(6+3i)-2i-i^2}{4+2i-2i-i^2}$$

Find zeros of each function by using the Quadratic Formula.

18. $y = 3x^2 - 4x - 1$

$$\frac{4 \pm \sqrt{4^2 - 4(3)(-1)}}{2(3)}$$

$$\begin{aligned}x = -6 \\x = 1\end{aligned}$$

$$\frac{4 \pm \sqrt{16+12}}{6} = \frac{4 \pm \sqrt{28}}{6}$$

$$\frac{4 \pm \sqrt{74}}{6} = \frac{4 \pm 2\sqrt{19}}{6}$$

$$\frac{2 \pm \sqrt{19}}{3}$$

20. $x^2 + 6x + 12$

$$\frac{-6 \pm \sqrt{16^2 - 4(1)(12)}}{2(1)}$$

$$\frac{-6 \pm \sqrt{36-48}}{2}$$

$$\frac{-6 \pm \sqrt{-12}}{2}$$

$$\frac{-6 \pm 2i\sqrt{3}}{2}$$

21. $x^2 + 4x + 10$

$$\frac{-4 \pm \sqrt{16-4(1)(10)}}{2(1)}$$

$$\frac{-4 \pm \sqrt{16-40}}{2}$$

$$\frac{-4 \pm \sqrt{-24}}{2}$$

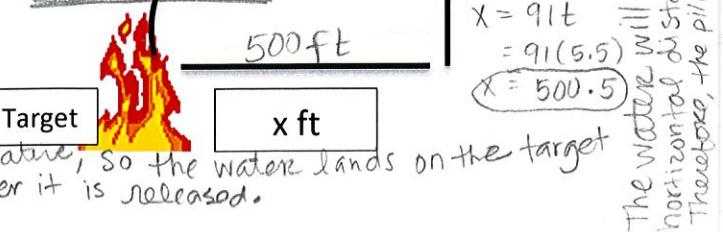
$$\frac{-4 \pm 2i\sqrt{6}}{2}$$

22. The pilot of a helicopter plans to release a bucket of water on a forest fire. The height y in feet of the water t seconds after its release is modeled by $y = -16t^2 - 2t + 500$. The horizontal distance x in feet between the water and its point of release is modeled by $x = 91t$. At what horizontal distance from the fire should the pilot start releasing the water in order to hit the target?

$$\frac{2 \pm \sqrt{2^2 - 4(-16)(500)}}{2(-16)} = \frac{2 \pm \sqrt{4+32000}}{-32} = \frac{2 \pm \sqrt{32004}}{-32}$$

$$\begin{aligned}t \geq 5.65 \\t \approx 5.53\end{aligned}$$

The time cannot be negative, so the water lands on the target about 5.5 seconds after it is released.



Release point

y

Path of water

x

Target

500 ft

x ft

The water will have traveled

horizontal distance, so about 500 ft.

Therefore, the pilot should start releasing the water when

Quadratic Quiz

Directions: Find the zeros of each function by using the Quadratic Formula.

$$1. \ x^2 + 6x$$

$$\begin{aligned} a &= 1 & -6 \pm \sqrt{6^2 - 4(1)(0)} \\ b &= 6 & 2(1) \\ c &= 0 & \\ \end{aligned}$$

a. 6, 1
 b. 6, 0
 c. **-6, 0**
 d. -6, 1

$$\frac{-6 \pm \sqrt{36 - 0}}{2} \Rightarrow \frac{-6 \pm 6}{2}$$

$$\boxed{x = -6}$$

$$\boxed{x = 0}$$

$$2. \ -x^2 - 2x + 9$$

$$\begin{aligned} a &= -1 & +2 \pm \sqrt{2^2 - 4(-1)(9)} \\ b &= -2 & 2(-1) \\ c &= 9 & \\ \end{aligned}$$

a. $-1 \pm \sqrt{5}$
 b. **$-1 \pm \sqrt{10}$**
 c. $-1 \pm \sqrt{15}$
 d. $-2 \pm \sqrt{10}$

$$\frac{2 \pm \sqrt{4 + 36}}{-2}$$

$$3. \ 7x^2 - 3$$

$$\begin{aligned} a &= 7 & \\ b &= 0 & \\ c &= -3 & \\ \end{aligned}$$

a. $\pm \frac{\sqrt{20}}{7}$
 b. $\pm \frac{\sqrt{21}}{12}$
 c. **$\pm \frac{\sqrt{21}}{7}$**
 d. $\pm \frac{\sqrt{19}}{7}$

$$\frac{0 \pm \sqrt{0^2 - 4(1)(-3)}}{2(1)} \Rightarrow \frac{\pm \sqrt{84}}{14} = \frac{\pm 2\sqrt{21}}{14}$$

$$\boxed{\pm \frac{\sqrt{21}}{7}}$$

$$4. \ -x^2 - x - 1$$

$$\begin{aligned} a &= -1 & +1 \pm \sqrt{1^2 - 4(-1)(-1)} \\ b &= -1 & 2(-1) \\ c &= -1 & \\ \end{aligned}$$

a. **$\frac{-1 \pm i\sqrt{3}}{2}$**
 b. $\frac{-1 \pm i\sqrt{5}}{2}$
 c. $\frac{-4 \pm i\sqrt{3}}{2}$
 d. $\frac{-1 \pm i\sqrt{3}}{6}$

$$\frac{1 \pm \sqrt{1 - 4}}{-2}$$

$$\frac{1 \pm \sqrt{3}}{-2}$$

$$\boxed{-\frac{1 \pm i\sqrt{3}}{2}}$$

$$5. \ 2x^2 + 7x - 13$$

$$\begin{aligned} a &= 2 & -7 \pm \sqrt{72 - 4(2)(-13)} \\ b &= 7 & 2(2) \\ c &= -13 & \\ \end{aligned}$$

a. $\frac{-8 \pm 3\sqrt{16}}{4}$
 b. $\frac{7 \pm 2\sqrt{17}}{4}$
 c. $\frac{-7 \pm 3\sqrt{17}}{4}$
 d. $\frac{-7 \pm 3\sqrt{17}}{6}$

$$6. \ -3x^2 + 4x - 4$$

$$\begin{aligned} a &= -3 & -4 \pm \sqrt{4^2 - 4(-3)(-4)} \\ b &= 4 & 2(-3) \\ c &= -4 & \\ \end{aligned}$$

a. $\frac{3 \pm 2i\sqrt{2}}{3}$
 b. $\frac{2 \pm 33i\sqrt{2}}{3}$
 c. $\frac{2 \pm 2i\sqrt{3}}{4}$
 d. $\frac{2 \pm 2i\sqrt{2}}{3}$

$$\frac{-4 \pm \sqrt{16 - 48}}{-6}$$

$$\frac{-4 \pm \sqrt{-32}}{-6} = \frac{\pm 4i\sqrt{2}}{-6}$$

$$\frac{-4 \pm 4i\sqrt{2}}{-6} = \boxed{\frac{2 \pm 2i\sqrt{2}}{3}}$$

hint
 hint
 cool
 problem

Egg Launch Contest

NAME: _____

DATE: _____

Cajun boy

Mr. Rhodes' class is holding an egg launching contest on the football field. Teams of students have built catapults that will hurl an egg down the field. Ms. Monroe's class will judge the contest. They have various tools and ideas for measuring each launch and how to determine which team wins.

Team A used their catapult and hurled an egg down the football field. Students used a motion detector to collect data while the egg was in the air. They came up with the table of data below.

DISTANCE FROM THE GOAL LINE (IN FEET)	HEIGHT (IN FEET)
7	19
12	90
14	101
19	90
21	55
24	0

$$y = a(x-h)^2 + k$$

$$y = a(x-14)^2 + 101$$

$$55 = a(21-14)^2 + 101$$

$$55 = a(7) + 101$$

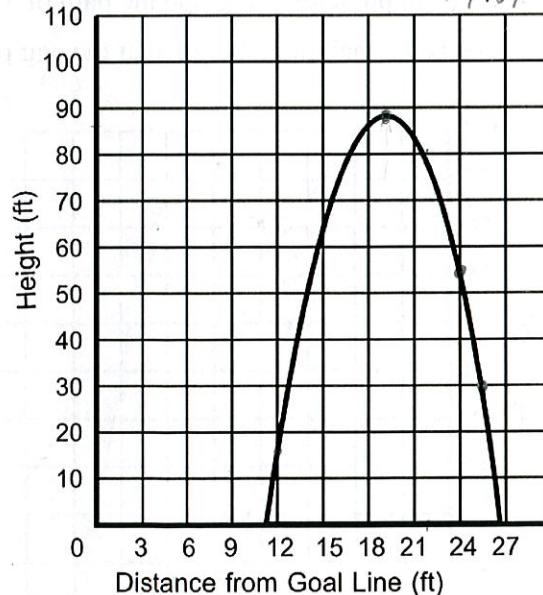
$$55 = 49a + 101$$

$$-46 = 49a \rightarrow a = -0.9387$$

Team B's egg flew through the air and landed down the field. The group of students tracking the path of the egg determined that the equation $y = -0.8x^2 + 19x - 40$ represents the path the egg took through the air, where x is the distance from the goal line and y is the height of the egg from the ground. (Both measures are in feet.)

When **Team C** launched an egg with their catapult, some of the judges found that the graph to the right shows the path of the egg.

Which team do you think won the contest?
Why?



$$90 = a(12-14)^2 + 101 \quad 19 = a(7-14)^2 + 101$$

$$90 = a(-2)^2 + 101 \quad 19 = a(-7)^2 + 101$$

$$90 = a4 + 101 \quad -101 = a49$$

$$-101 = -4a \quad -82 = 49a$$

$$\frac{-101}{-4} = a \quad \frac{-82}{49} = a$$

$$-2.525 = a \quad -1.67 = a$$

$$90 = a(12-14)^2 + 101$$

$$-.9387(x-14)(x-14) + 100$$

$$-.9387(x^2 - 14x - 14x + 196) + 100$$

$$(-x^2 - 28x + 196) + 100$$

Team A

- Using the data from Team A, determine an equation that describes the path of the egg. Describe how you found your equation.
- On the graph below, graph the path of Team A's egg.
- What is the maximum height that the egg reached? How far was the egg hurled?

101 feet

Team B

- Using the equation from Team B, generate a table of values that shows different locations of the egg as it flew through the air.

x	10	15	20	12	14	13	11	3	2.33	21.4	
y	70	65	20	72.8	69.2	71.8	72.2	29.8	0	0	

$$y = -0.8x^2 + 19x - 40$$

- On the graph below, graph the path of Team B's egg.

$$x = \frac{-19 \pm \sqrt{19^2 - 4(-0.8)(-10)}}{2(-0.8)}$$

- What is the maximum height that the egg reached? How far was the egg hurled?

72.8 feet

21.4 feet

$\sqrt{361 - 128}$

(19, 0)

-1.6

Team C

- Using the data from Team C, generate a table of values that shows different locations of the egg as it flew through the air.

x	11	12	13	15	17	20	22	24	25	26	
y	0	15	30	63	80	99	80	55	30	0	

- On the graph below, re-graph the path of Team C's egg.

- What is the maximum height that the egg reached? How far was the egg hurled?

~89 feet

~26 feet

15 feet

- If it is a height contest, which team wins? How do you know?

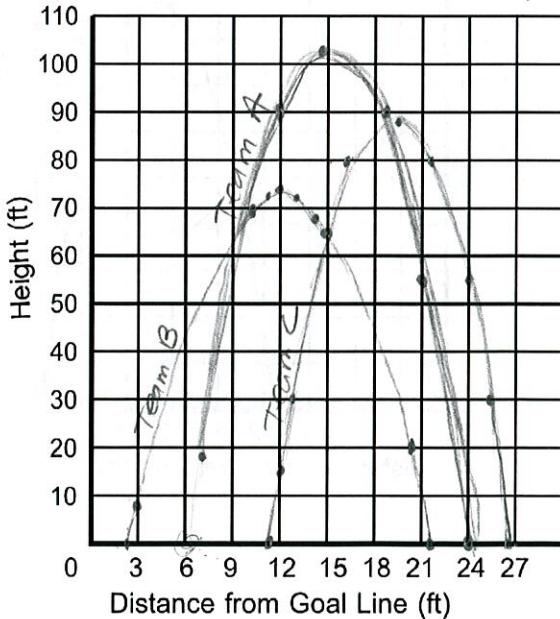
Team A the y-value in the middle is the biggest.

- If it is a distance contest, which team wins? How do you know?

Team B the distance from beginning to end is more.

- Find a method of determining a winner so that the team that did not win in Question 10 or Question 11 would win using your method.

19.07



Cognitoy

Practice:

Identify the degree of each monomial.

1. x^3

The degree
is 3

2. 7

$7 = 7x^0$
degree is 0.

3. $5x^3y^2$

$5x^3y^2 = 5$
degree is
5.

4. a^6bc^2

degree is
9.

Rewrite each polynomial in standard form. Then identify the leading coefficient, degree, and number of terms. Name the polynomial.

5. $4x - 2x^2 + 2$

$$-2x^2 + 4x + 2$$

LC: -2

degree: 2

of terms: 3

Name: quadratic trinomial

6. $-18x^2 + x^3 - 5 + 2x$

$$x^3 - 18x^2 + 2x - 5$$

LC: 1

degree: 3

of terms: 4

Name: cubic polynomial
w/ 4 terms.

Add or subtract. Write your answer in standard form.

7. $(-36x^2 + 6x - 11) + (6x^2 + 16x^3 - 5)$

$$-36x^2 + 6x - 11$$

$$+ 16x^3 + 6x^2 + 0x - 5$$

$$\boxed{16x^3 - 30x^2 + 6x - 16}$$

8. $(5x^3 + 12 + 6x^2) - (15x^2 + 3x - 2)$

$$(5x^3 + 6x^2 + 12) + (-15x^2 - 3x + 2)$$

$$(5x^3) + (6x^2 - 15x^2) + (-3x) + (12 + 2)$$

$$\boxed{5x^3 - 9x^2 - 3x + 14}$$

9. For different patient, the dye dilution can be modeled by the function $f(t) = 0.000468t^4 - 0.016t^3 + 0.095t^2 + 0.806t$. Evaluate $f(t)$ for $t = 4$ and $t = 17$, and describe what the values of the function represent.

$$f(4) = 0.000468(4)^4 - 0.016(4)^3 + 0.095(4)^2 + 0.806(4) \Rightarrow 3.8398$$

The concentration of dye 4 seconds after the injection is 3.8398 mg/L.

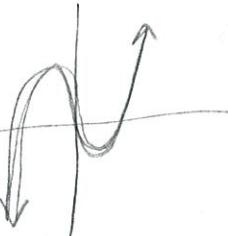
$$f(17) = 0.000468(17)^4 - 0.016(17)^3 + 0.095(17)^2 + 0.806(17) \Rightarrow 1.6368$$

The concentration of dye 17 seconds after the injection is 1.6368 mg/L.

Graph each polynomial function on a calculator. Describe the graph, and identify the number of real zeros.

10. $f(x) = 6x^3 + x^2 - 5x + 1$

increases, decreases slightly, & increases again. crosses x-axis 3 times appear to be 3 real zeros.



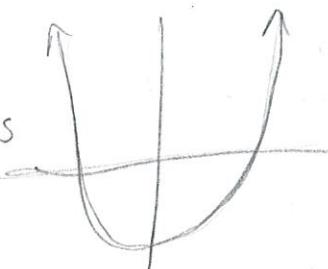
11. $f(x) = 3x^2 - 2x + 2$

decreases then increases. not cross x-axis no real zeros.



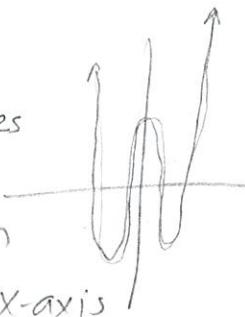
12. $g(x) = x^4 - 3$

decreases then increases, crosses x-axis twice, appear to be 2 real zeros.



13. $h(x) = 4x^4 - 16x^2 + 5$

alternately decreases and increases change direction 3 times. crosses x-axis 4 times. appear 4 real zeros.



GET ORGANIZED!!

Characteristics

Definition

Polynomial

Examples

Nonexamples

Multiplying Polynomials

Who uses this?

Business managers can multiply polynomials when modeling total manufacturing costs.



To multiply a polynomial by a monomial, use the Distributive Property and the Properties of Exponents.

Example 1 - Multiplying a Monomial and a Polynomial

Find each product.

A. $3x^2(x^3 + 4)$

$$3x^5 + 12x^2$$

B. $ab(a^3 + 3ab^2 - b^3)$

$$a^4b + 3a^2b^3 - ab^4$$

To multiply any two polynomials, use the Distributive Property and multiply each term in the second polynomial by each term in the first.

$$(x + 2)(x^2 + 4x - 3)$$

Keep in mind that if one polynomial has m terms and the other has n terms, than the product has mn terms before it is simplified.

Example 2 – Multiplying Polynomials

Find each product.

A. $(x - 2)(1 + 3x - x^2)$

$$x + 3x^2 - x^3 - 2 - 6x + 2x^2$$

$$-x^3 + 5x^2 - 5x - 2$$

B. $(x^2 + 3x - 5)(x^2 - x + 1)$

$$x^4 - x^3 + x^2 + 3x^3 - 3x^2 + 3x - 5x^2 + 5x - 5$$

$$x^4 + 2x^3 - 7x^2 + 8x - 5$$

x^2	$-x$	$+$	
x^2	x^4	$-x^3$	$+x^2$
$+3x$	$+3x^3$	$-3x^2$	$+3x$
-5	$-5x^2$	$+5x$	-5

$$-4 \in -5 \Rightarrow -0.0004$$

Example 3 – Business Application

Mr. Silva manages a manufacturing plant. From 1990 through 2005, the number of units produced (in thousands) can be modeled by $N(x) = 0.02x^2 + 0.2x + 3$. The average cost per unit (in dollars) can be modeled by $C(x) = -0.002x^2 - 0.1x + 2$, where x is the number of years since 1990. Write a polynomial $T(x)$, that can be used to model Mr. Silva's total manufacturing costs.

$$T(x) = N(x) \cdot C(x)$$

$$T(x) = (0.02x^2 + 0.2x + 3)(-0.002x^2 - 0.1x + 2)$$

$$= -0.0004x^4 - 0.002x^3 + .04x^2 - 0.0004x^3 - .02x^2 + .4x$$

$$= -.0004x^4 - .3x^3 + .04x^2 - .0004x^3 - .02x^2 + .4x$$

$$T(x) = -0.0004x^4 - 0.0024x^3 + .014x^2 + 0.1x + 6$$

You can also raise polynomials to powers. Mr. Silva's total manufacturing costs, in thousands of dollars, can be modeled by

Example 4 – Expanding a Power a Binomial

Find the product.

$$(x+y)^3$$

$$(x+y)(x+y)(x+y)$$

$$(x+y)(x^2 + xy + xy + y^2)$$

$$(x+y)(x^2 + 2xy + y^2)$$

$$x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$$

0	$-0.0004x^4$	$.002x^3$	$.04x^2$
1	$-0.0004x^3$	$.02x^2$	$.4x$
2	$-0.0004x^2$	$.3x$	6

$$x^3 + 3x^2y + 3xy^2 + y^3$$

Notice the coefficients of the variables in the final product of $(x+y)^3$. These coefficients are the numbers from the third row of Pascal's triangle.

Binomial Expansion		Pascal's Triangle (Coefficients)
$(a+b)^0 =$	1	1
$(a+b)^1 =$	$a+b$	1 1
$(a+b)^2 =$	$a^2 + 2ab + b^2$	1 2 1
$(a+b)^3 =$	$a^3 + 3a^2b + 3ab^2 + b^3$	1 3 3 1
$(a+b)^4 =$	$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	1 4 6 4 1
$(a+b)^5 =$	$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$	1 5 10 10 5 1
$(a+b)^6 =$	$a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$	1 6 15 20 15 6 1
$(a+b)^7 =$	$a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + b^7$	1 7 21 35 35 21 7 1

Each row of Pascal's triangle gives the coefficients of the corresponding binomial expansion. The pattern in the table can be extended to apply to the expansion of any binomial of the form $(a+b)^n$, where n is a whole number.

Binomial Expansion

For a binomial expansion of the form $(a + b)^n$, the following statements are true.

1. There are $n + 1$ terms.
2. The coefficients are the numbers from the n th row of Pascal's triangle.
3. The exponent of a is n in the first term, and the exponent decreases by 1 in each successive term.
4. The exponent of b is 0 in the first term, and the exponent increases by 1 in each successive term.
5. The sum of the exponents in any term is n .

Example 5 – Using Pascal's Triangle to Expand Binomial Expressions

Expand each expression.

A. $(y - 3)^4$

1 4 6 4 | Identify row

$$[1y^4(-3)^0] + [4y^3(-3)^1] + [6y^2(-3)^2] + [4y^1(-3)^3] + [1y^0(-3)^4]$$
$$(y^4 - 12y^3 + 54y^2 - 108y + 81)$$

1 3 3 1 - Row

$$[(4z)^3(5)^0] + [3(4z)^2(5)^1] + [3(4z)^1(5)^2]$$

B. $(4z + 5)^3$

$$[(4z)^0(5)^3]$$

$$(64z^3 + 240z^2 + 300z + 125)$$

GET ORGANIZED: In each box, write an example and find the product.

Binomial x Trinomial
(horizontal method)

$$(x+3)(x^2+4x-6)$$

$$x^3 + 4x^2 - 6x + 3x^3 + 12x - 18$$

$$x^3 + 7x^2 + 6x - 18$$

X	Y
x^3	x^2y
$3x^2y$	$3xy^2$
$5y$	$5y^3$

$$(x+y)(x^2+3xy+5y)$$

Binomial x Trinomial
(vertical method)

$$x^3 + 4x^2y + 8xy^2 + 5y^3$$

Monomial x Trinomial

$$2x(x^3+x^2-4)$$

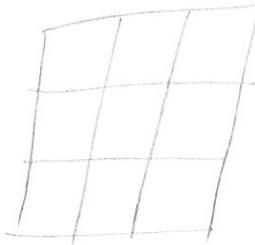
Multiplying Polynomials

$$(x^3+3x+3)(x^2+3x+2)$$

Trinomial x Trinomial

Expand a Binomial

$$(2a-4)^5$$



Practice:

Find each product.

$$1. \quad 3cd^2(4c^2d - 6cd + 14cd^2)$$

$$12c^3d^3 - 18c^2d^3 + 42c^2d^4$$

$$2. \quad x^2y(6y^3 + y^2 - 28y + 30)$$

$$6x^2y^4 + x^2y^3 - 28x^2y^2 + 30x^2y$$

Find each product.

$$3. \quad (3b - 2c)(3b^2 - bc - 2c^2)$$

$$\begin{array}{r} 3b \quad -2c \\ \hline 3b^2 \quad 9b^3 \quad -6b^2c \\ -bc \quad -3b^2c \quad 2bc^2 \\ -2c^2 \quad -4bc^2 \quad 4c^3 \end{array}$$

$$[9b^3 - 9b^2c - 4bc^2 + 4c^3]$$

$$4. \quad (x^2 - 4x + 1)(x^2 + 5x - 2)$$

$$\begin{array}{r} x^2 \quad -4x \quad 1 \\ \hline x^4 \quad -4x^3 \quad x^2 \\ 5x \quad 5x^3 \quad -20x^2 \quad 5x \\ -2 \quad -2x^2 \quad 8x \quad -2 \end{array}$$

$$x^4 + x^3 - 21x^2 + 13x - 2$$

5. **What if...?** Suppose that in 2005 the cost of raw materials increases and the new average cost per unit is modeled by $C(x) = -0.004x^2 - 0.1x + 3$. Write a polynomial $T(x)$ that can be used to model the total costs.

$$N(x) = 0.02x^2 + 0.2x + 3 \Rightarrow \text{number of units}$$

$$-0.004x^2 - 0.1x \quad 3$$

$$\begin{array}{r} 0.02x \quad -0.0008x^4 \quad 0.002x^3 \quad 0.06x^2 \\ 0.2x \quad -0.0008x^3 \quad -0.02x^2 \quad .6x \\ 3 \quad -.012x^2 \quad -.3x \quad 9 \end{array}$$

$$T(x) = -0.00008x^4 - 0.0028x^3 + 0.028x^2$$

$$.3x + 9$$

Find each product.

$$6. \quad (x+4)^4$$

$$(x+4)(x+4)(x+4)(x+4)$$

$$(x^2 + 4x + 4x + 16)$$

$$(x^2 + 8x + 16)(x^2 + 8x + 16)$$

$$x^4 + 8x^3 + 16x^2 + 8x^3 + 64x^2 + 128x + 16x^2 + 128x + 256$$

$$8x^3 - 12x^2 + 6x - 1$$

$$7. \quad (2x-1)^3$$

$$(2x-1)(2x-1)(2x-1)$$

$$(4x^2 - 2x - 2x + 1)$$

$$(4x^2 - 4x + 1)(2x-1)$$

$$\begin{array}{r} 2x \quad -1 \\ 4x^2 \quad 8x^3 \quad -4x^2 \\ -4x \quad -8x^2 \quad 4x \\ 1 \quad 2x \quad -1 \end{array}$$

Expand each expression.

$$8. \quad (x+2)^3$$

$$1 \ 3 \ 3 \ 1$$

$$[(1(x)^3(2)^0) + [3(x)^2(2)^1] + [(3(x)^1(2)^2)] + [1(x)^0(2)^3]]$$

$$9. \quad (x-4)^5$$

$$1 \ 5 \ 10 \ 10 \ 5 \ 1$$

$$[(1(x)^5(4)^0) + [5(x)^4(4)^1] + [10(x)^3(4)^2] + [10(x)^2(4)^3] + [5(x)^1(4)^4] + [1(x)^0(4)^5]]$$

$$10. \quad (3x+1)^4$$

$$1 \ 4 \ 6 \ 4 \ 1$$

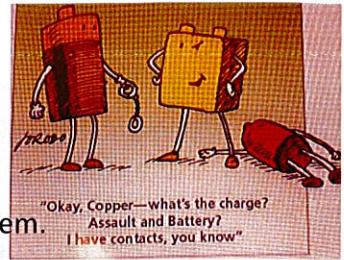
$$x^5 - 20x^4 + 160x^3 - 640x^2 + 1280x - 1024$$

Cajuday's notes

Dividing Polynomials

Who uses this?

Electricians can divide polynomials in order to find the voltage in an electrical system.



Polynomial long division is a method for dividing a polynomial by another polynomial of a lower degree. It is very similar to dividing numbers.

Arithmetic Long Division

$$\begin{array}{r} \text{Divisor} \longrightarrow 4 \overline{)692} \\ \quad \quad \quad \text{Quotient} \quad \leftarrow 173 \\ \quad \quad \quad \text{Dividend} \\ -4 \\ \hline 29 \\ -28 \\ \hline 12 \\ -12 \\ \hline 0 \quad \leftarrow \text{Remainder} \end{array}$$

Polynomial Long Division

$$\begin{array}{r} 3x^2 - 3x - 1 \\ \hline x + 1 \quad \boxed{3x^3 + 0x^2 - 4x - 1} \\ \quad \quad \quad (-3x^3 + 3x^2) \\ \hline \quad \quad \quad -3x^2 - 4x - 1 \\ \quad \quad \quad (+3x^2 + 3x) \\ \hline \quad \quad \quad -x - 1 \\ \quad \quad \quad (-x - 1) \\ \hline 0 \end{array}$$

Example 1: Using Long Division to Divide Polynomials

Divide by using long division.

$$(4x^2 + 3x^3 + 10) \div (x - 2)$$

Step 1: Write the dividend in standard form, including terms with a coefficient of 0.

$$3x^3 + 4x^2 + 0x + 10$$

Step 2: Write division in the same way as you would when dividing numbers.

$$x - 2 \overline{)3x^3 + 4x^2 + 0x + 10}$$

Step 3: Divide.

$$\begin{array}{r} 3x^2 + 10x + 20 \\ \hline x - 2 \overline{)3x^3 + 4x^2 + 0x + 10} \\ - (3x^3 - 6x^2) \downarrow \\ \quad \quad \quad 10x^2 + 0x \\ - (10x^2 - 20x) \downarrow \\ \quad \quad \quad 20x + 10 \\ - (20x - 40) \\ \quad \quad \quad 50 \end{array}$$

Step 4: Write the final answer.

$$\frac{4x^2 + 3x^3 + 10}{x - 2} = 3x^2 + 10x + 20 + \frac{50}{x - 2}$$

Synthetic division is a shorthand method of dividing a polynomial by a linear binomial by using only the coefficients. For synthetic division to work, the polynomial must be written in standard form, using 0 as a coefficient for any missing terms, and the divisor must be in the form $(x - a)$.

Synthetic Division Method

Divide $(2x^2 + 7x + 9) \div (x + 2)$ by using synthetic division.

WORDS	NUMBERS
Step 1 Write the coefficients of the dividend, 2, 7, and 9. In the upper left corner, write the value of a for the divisor $(x - a)$. So $a = -2$. Copy the first coefficient in the dividend below the horizontal bar.	$\begin{array}{r} -2 2 & 7 & 9 \\ \hline & 2 \end{array}$
Step 2 Multiply the first coefficient by the divisor, and write the product under the next coefficient. Add the numbers in the new column.	$\begin{array}{r} -2 2 & 7 & 9 \\ & -4 \downarrow & \\ & 2 \end{array}$
Repeat Step 2 until additions have been completed in all columns. Draw a box around the last sum.	$\begin{array}{r} -2 2 & 7 & 9 \\ & -4 & -6 \\ \hline & 2 & 3 \boxed{3} \end{array}$
Step 3 The quotient is represented by the numbers below the horizontal bar. The boxed number is the remainder. The others are the coefficients of the polynomial quotient, in order of decreasing degree.	$= 2x + 3 + \frac{3}{x+2}$

Example 2: Using Synthetic Division to Divide by a Linear Binomial

Divide by using synthetic division.

A. $(4x^2 - 12x + 9) \div (x + \frac{1}{2})$

B. $(x^4 - 2x^3 + 3x + 1) \div (x - 3)$

Step 1: find a , Then write the coefficients and a in the synthetic division format.

$$a = -\frac{1}{2}$$

$$\begin{array}{r} -\frac{1}{2} | 4 & -12 & 9 \\ \hline & 4 & -2 & 7 \\ & \downarrow & \downarrow & \downarrow \\ & 4 & -14 & \boxed{16} \end{array}$$

$$a = 3$$

$$\begin{array}{r} 3 | 1 & -2 & 0 & 3 & 1 \\ & 3 & 3 & 9 & 36 \\ \hline & 1 & 3 & 12 & \boxed{37} \end{array}$$

Step 2: Bring down the first coefficient. Then multiply and add for each column.

Step 3: Write the quotient.

$$x^3 + x^2 + 3x + 12 + \frac{37}{x-3}$$

$$4x - 14 + \frac{16}{x+\frac{1}{2}}$$

You can use synthetic division to evaluate polynomials. This process is called synthetic substitution. The process of synthetic division, but the final answer is interpreted differently, as described by the Remainder Theorem.

Remainder Theorem	
THEOREM	EXAMPLE
If the polynomial function $P(x)$ is divided by $x - a$, then the remainder r is $P(a)$.	Divide $x^3 - 4x^2 + 5x + 1$ by $x - 3$. $\begin{array}{r} 3 1 & -4 & 5 & 1 \\ & 3 & -3 & 6 \\ \hline & 1 & -1 & 2 & 7 \end{array}$ $P(3) = 7$

Example 3: Using Synthetic Substitution

Using synthetic substitution to evaluate the polynomial for the given value.

A. $P(x) = x^3 - 4x^2 + 3x - 5$ for $x = 4$

$$\begin{array}{r} 4 | 1 & -4 & 3 & -5 \\ & 4 & 0 & 12 \\ \hline & 1 & 0 & 3 & | 7 \end{array}$$

$$P(4) = 7$$

$$\begin{aligned} P(4) &= (4)^3 - 4(4)^2 + 3(4) - 5 \\ &= 64 - 64 + 12 - 5 \\ &= 7 \end{aligned}$$

B. $P(x) = 4x^4 + 2x^3 + 3x + 5$ for $x = -\frac{1}{2}$

$$\begin{array}{r} -\frac{1}{2} | 4 & 2 & 3 & 5 \\ & -2 & 0 & -1.5 \\ \hline & 4 & 0 & 3 & | 3.5 \end{array}$$

$$P(-\frac{1}{2}) = 3.5$$

$$\begin{aligned} P(-\frac{1}{2}) &= 4(-\frac{1}{2})^4 + 2(-\frac{1}{2})^3 + 3(-\frac{1}{2}) + 5 \\ &= .25 - .25 - 1.5 + 5 \\ &= 3.5 \end{aligned}$$

Example 4: Physics Application

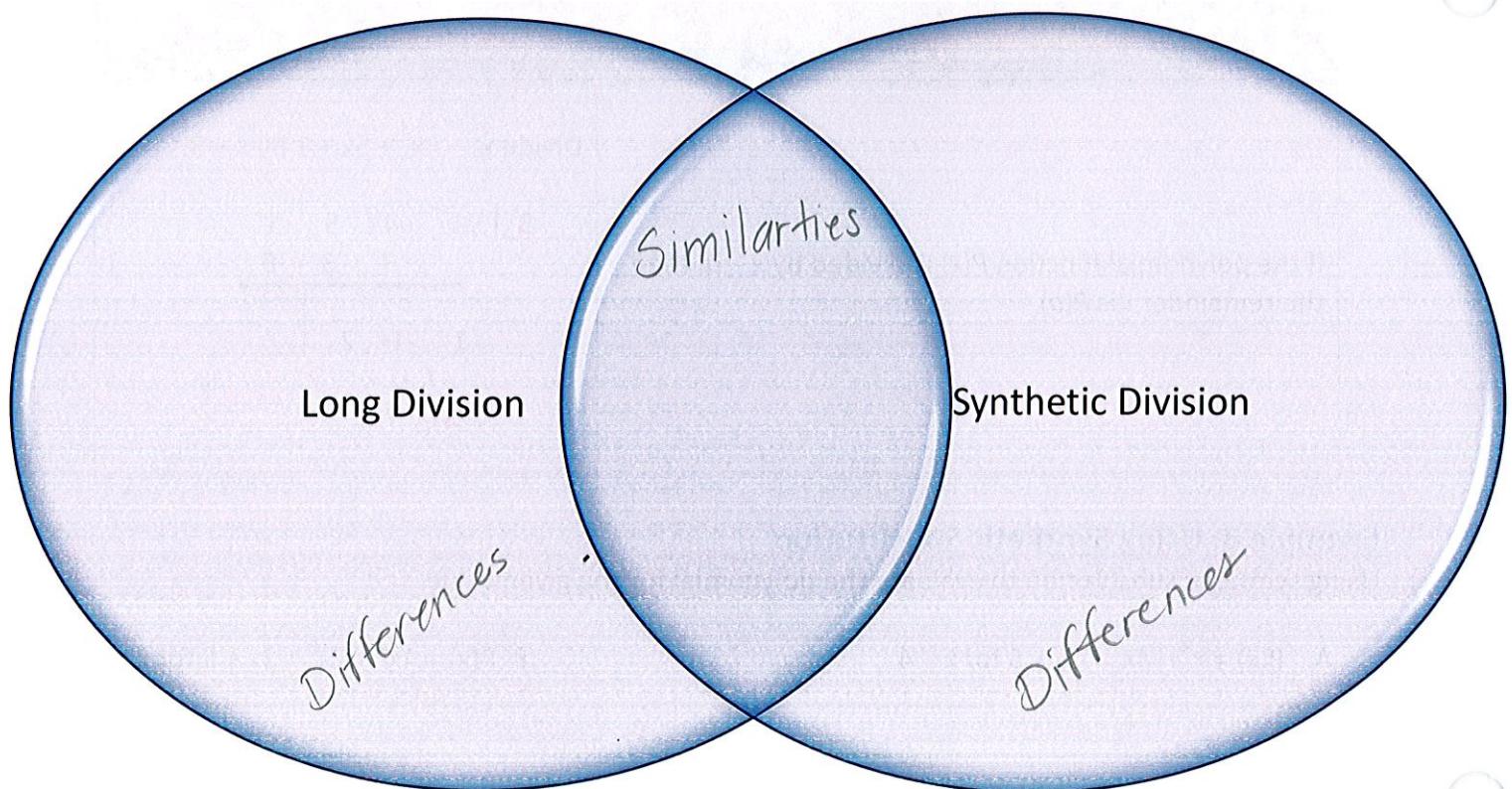
A Van de Graaff generator is a machine that produces very high voltages by using small, safe levels of electric current. One machine has a current that can be modeled by $I(t) = t + 2$, where $t > 0$ represents time in seconds. The power of the system can be modeled by $P(t) = 0.5t^3 + 6t^2 + 10t$. Write an expression that represents the voltage of the system.

$$V(t) = \frac{0.5t^3 + 6t^2 + 10t}{t + 2}$$

$$\begin{array}{r} -2 | 0.5 & 6 & 10 & 0 \\ & -1 & -10 & 0 \\ \hline & 0.5 & 5 & 0 & | 0 \end{array}$$

$$V(t) = 0.5t^2 + 5t$$

GET ORGANIZED - Complete the graphic organizer.



Cajudoy's notes

Dividing Polynomials - Practice:

Divide by using long division.

1. $(15x^2 + 8x - 12) \div (3x + 1)$

$$\begin{array}{r} 5x + 1 \\ \hline 3x+1 \longdiv{15x^2 + 8x - 12} \\ -(15x^2 + 5x) \downarrow \\ 3x - 12 \\ -(3x + 1) \\ \hline -13 \end{array}$$

$5x + 1 - \frac{13}{3x+1}$

Divide by using synthetic division.

3. $(6x^2 - 5x - 6) \div (x + 3)$

$$\begin{array}{r} 6 -5 -6 \\ \downarrow -18 69 \\ \hline 6 -23 63 \end{array}$$

$6x - 23 + \frac{63}{x+3}$

Use synthetic substitution to evaluate the polynomial for the given value.

5. $P(x) = x^3 + 3x^2 + 4$ for $x = -3$

$$\begin{aligned} P(-3) &= (-3)^3 + 3(-3)^2 + 4 \\ &= -27 + 27 + 4 \end{aligned}$$

$P(-3) = 4$

6. $P(x) = 5x^2 + 9x + 3$ for $x = \frac{1}{5}$

$$\begin{aligned} P\left(\frac{1}{5}\right) &= P(0.2) \\ &= 5(0.2)^2 + 9(0.2) + 3 \\ &= 0.2 + 1.8 + 3 \end{aligned}$$

$P\left(\frac{1}{5}\right) = 5$

7. Write an expression for the length of a rectangle with width $y - 9$ and area $y^2 - 14y + 45$.

$$l = \frac{y^2 - 14y + 45}{y - 9}$$

$$\begin{array}{r} 1 -14 45 \\ 9 -45 \\ \hline 1 -5 0 \end{array}$$

$l = y - 5$

Factoring Polynomials

Who uses this?

Ecologists may use factoring polynomials to determine when species might become extinct.



Recall that if a number is divided by any of its factors, the remainder is 0. Likewise, if a polynomial is divided by any of its factors, the remainder is 0.

The Remainder Theorem states that if a polynomial is divided by $(x - a)$, the remainder is the value of the function at a . So, if $(x - a)$ is a factor of $P(x)$, then $P(a) = 0$.

Factor Theorem

Theorem	Example
For any polynomial $P(x)$, $(x - a)$ is a factor of $P(x)$ if and only if $P(a) = 0$.	Because $P(1) = 1^2 - 1 = 0$, $(x - 1)$ is a factor of $P(x) = x^2 - 1$.

Example 1: Determining Whether a Linear Binomial is a Factor

Determine whether the given binomial is a factor of the polynomial $P(x)$.

A. $(x - 3)$; $P(x) = x^2 + 2x - 3$

Find $P(3)$

$$\begin{array}{r} 3 | \quad 1 \quad 2 \quad -3 \\ \downarrow \quad 3 \quad 15 \\ \hline 1 \quad 5 \quad |12 \end{array}$$

$$P(3) = 12$$

$P(3) \neq 0$, so $(x - 3)$

is not a factor
of $P(x) = x^2 + 2x - 3$

B. $(x + 4)$; $P(x) = 2x^4 + 8x^3 + 2x + 8$

$$\begin{array}{r} x^2 \\ P(-4) \\ -4 | \quad 2 \quad 8 \quad 0 \quad 2 \quad 8 \\ \downarrow \quad -8 \quad 0 \quad 0 \quad -8 \\ \hline 2 \quad 0 \quad 0 \quad 2 \quad |0 \end{array}$$

$P(-4) = 0$, so $(x + 4)$ is a factor of $P(x)$

- You are already familiar with methods for factoring quadratics expressions. You can factor polynomials of higher degrees using many of the same methods you previously learned.

Example 2: Factoring by Group

Factor $x^3 + 3x^2 - 4x - 12$.

$$(x^3 + 3x^2) + (-4x - 12)$$

$$x^2(x + 3) - 4(x + 3)$$

$$(x + 3)(x^2 - 4)$$

$$(x + 2)(x - 2)$$

$$(x + 3)(x + 2)(x - 2)$$

Just as there is a special rule for factoring the difference of two squares, there are special rules for factoring the sum or difference of two cubes.

Factoring the Sum and the Difference of Two Cubes

METHOD	ALGEBRA
Sum of two cubes	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
Difference of two cubes	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Example 3: Factoring the Sum or Difference of Two Cubes

Factor each expression.

A. $5x^4 + 40x$

$$5x(x^3 + 8)$$

$$5x(x^3 + 2^3)$$

$$5x(x+2)(x^2 - x \cdot 2 + 2^2)$$

$$\boxed{5x(x+2)(x^2 - 2x + 4)}$$

B. $8y^3 - 27$

$$(2y)^3 - 3^3$$

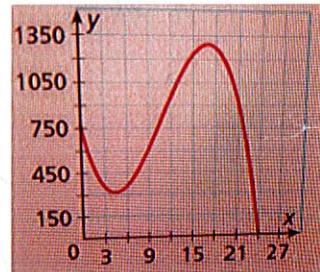
$$(2y-3)[(2y)^2 + 2y \cdot 3 + 3^2]$$

$$\boxed{(2y-3)(4y^2 + 6y + 9)}$$

You can also use a graph to help you factor a polynomial. Recall that the real zeros of a function appear as x-intercepts on its graph. By the Factor Theorem, if you can determine the zeros of a polynomial function from its graph, you can determine the corresponding factors of the polynomial.

Example 4: Ecology Application

The population of an endangered species of bird in the years since 1990 can be modeled by the function $P(x) = -x^3 + 32x^2 - 224x + 768$. Identify the year that the bird will become extinct if the model is accurate and no protective measures are taken. Use the graph to factor $P(x)$.



where does $y = 0$

The zero is $x = 24 = 2014$

($x - 24$)

$$\begin{array}{r} 24 | -1 \ 32 \ -224 \ 768 \\ \quad \downarrow -24 \quad 192 \ -768 \\ \hline -1 \ 8 \ -32 \ 0 \end{array}$$

$$P(x) = (x-24)(x^2 + 8x - 32)$$

factor -1

$$P(x) = -(x-24)(x^2 - 8x + 32)$$

GET ORGANIZED: For each method, give an example of a polynomial and its factored form.

Method	Polynomial	Factored Form
Difference of Two Squares	$x^2 - 4$	$(x+2)(x-2)$
Difference of Two Cubes		
Sum of Two Cubes		